Algorithm for Rotation Measure Synthesis

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This document has been written for the ASKAP computation team. It describes the algorithm that the POSSUM team has been using to perform Rotation Measure (RM) Synthesis. I've attached an IDL file named `rmsynthesis.pro`, which contains a well-commented implementation of the algorithm described below. The IDL language is very similar to fortran such that the algorithm and comments should be easily translated to fortran (I do not know fortran and therefore cannot provide a fortran implementation of this algorithm.)

This document does not describe the RM Clean step. In the parlance of RM Synthesis (as described below), this algorithm produces a dirty Faraday dispersion function. Detailed exegeses of the RM Synthesis method can be found in Brentjens & de Bruyn (2005) and Heald (2009).

I start by defining a few quantities. Burn (1966) defines the **Faraday depth** to be

$$\phi(\mathbf{r})[\mathrm{rad\ m^{-2}}] = 0.81 \int_{there}^{here} n_{\mathrm{e}} \mathbf{B}[\mu\mathrm{G}] \cdot \mathbf{dr}[\mathrm{pc}]\,, \tag{1}$$

where n_{e} is the electron density in $\mathrm{cm^{-3}}$, $\mathbf{B}$ is the magnetic field in microgauss, $\mathbf{dr}$ is the infinetesimal path length in parsecs, the integral is carried out from the source to the observer, and both $\phi > 0$ and $B > 0$ imply a magnetic field pointing towards the observer.[1] If the radiation from an emitting source is intrinsically polarized with an angle of χ_0, then the propagation of the polarized emission through a Faraday depth $\phi(r)$ will cause the angle of polarization to be rotated to $\chi(r, \lambda) = \chi_0(r) + \phi(r)\lambda^2$. Burn (1966) defines the Fourier transform of the complex polarized intensity to be the "**Faraday dispersion function**" (FDF),[2] given by eq. (25) of Brentjens & de Bruyn (2005):

$$\tilde{F}(\phi) = K \int_{-\infty}^{+\infty} \tilde{P}(\lambda^2) e^{-2i\phi(\lambda^2 - \lambda_0^2)} d\lambda^2\,, \tag{2}$$

where $\tilde{P}(\lambda^2)$ is the product of the observed complex polarized surface brightness and a windowing function $W(\lambda^2)$, λ_0^2 is the weighted average of the square of the observed wavelength,

[1] This definition is opposite to the convention in Zeeman splitting, where a positive field (like a Doppler velocity) points away from observer.

[2] Note that the FDF is a complex quantity. The polarized intensity is given by the amplitude of the complex FDF.

and K is a normalization factor that is the inverse of the integral of the windowing function:

$$K = \left(\int_{-\infty}^{+\infty} W(\lambda^2) d\lambda^2 \right)^{-1} . \tag{3}$$

The wavelength-squared sampling of the data is irregular and incomplete: this produces sidelobes (analogous to those produced in aperture synthesis) in the FDF response. This response function (a complex quantity) is known as the **rotation measure spread function** (RMSF)[3] and is defined as

$$R(\phi) = K \int_{-\infty}^{+\infty} W(\lambda^2) e^{-2i\phi(\lambda^2 - \lambda_0^2)} d\lambda^2 . \tag{4}$$

The FWHM of the main peak of the RMSF is estimated by Brentjens & de Bruyn (2005) to be $\delta\phi \approx 2\sqrt{3}/\Delta(\lambda^2)$, where $\Delta(\lambda^2) = \lambda^2_{\rm max} - \lambda^2_{\rm min}$ is the total bandwidth of the observations in wavelength squared. Because we can quantify this response function, we can use Hogbom's cleaning method to deconvolve the RMSF from our data.

In practice, we carry out the RM Synthesis scheme in the following way. First, a complex polarization image is created at each observed frequency from the Stokes Q and U images (real quantities), yielding a complex polarization cube $P(x, y, \nu) = (Q(x, y, \nu), U(x, y, \nu))$. We choose a Faraday depth sampling that is much smaller than the FWHM of the RMSF,[4] and at each spatial position we populate the FDF at each Faraday depth channel j using the equation:

$$\tilde{F}(x, y, \phi_j) = K \sum_{i=1}^{N} w_i P_i(x, y) e^{-2i\phi_j(\lambda_i^2 - \lambda_0^2)} , \tag{5}$$

where the summation is over frequency channels and

$$K = \left(\sum_{i=1}^{N} w_i \right)^{-1} , \tag{6}$$

and

$$\lambda_0^2 = K \sum_{i=1}^{N} w_i \lambda_i^2 . \tag{7}$$

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[3]Brentjens & de Bruyn (2005) introduced this quantity as the rotation measure transfer function (RMTF), but it was renamed by Heald (2009) because, in anology to telescope optics, the quantity is more closely related to a point spread function than an optical transfer function.

[4]A critical sampling of the FDF involves 2–3 Faraday depth points intervals per $\delta\phi$ (Brentjens & de Bruyn 2005). To determine the peaks accurately, one should sample many dozen points per $\delta\phi$. Our sampling is equivalent to $\sim$100 Faraday depth intervals per $\delta\phi$.

We use a uniform weighting such that $w_i = 1$ and $K = N^{-1}$.

The quantity $\tilde{F}(\phi)$ is what we're after. It's what we refer to as the dirty Faraday dispersion function cube or dirty FDF cube. It is a complex quantity. IDL can store complex data. As I understand it, fortran cannot, so the real and imaginary components of the dirty FDF must be stored as the output from RM Synthesis.

We don't need the RMSF at this stage. We will need it when we eventually run the RM Clean algorithm. The RMSF will be given by:

$$R(\phi_j) = K \sum_{i=1}^{N} w_i e^{-2i\phi_j(\lambda_i^2 - \lambda_0^2)} \,. \tag{8}$$

REFERENCES

Brentjens, M. A., & de Bruyn, A. G. 2005, A&A, 441, 1217

Burn, B. J. 1966, MNRAS, 133, 67

Heald, G. 2009, in IAU Symposium 259, IAU Symposium, 591