POSSUM Report #17:
Tabulation of Options for RMCLEAN Algorithm
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1. INTRODUCTION

This report describes the currently known options for cleaning dirty Faraday dispersion functions produced by the Rotation Measure (RM) synthesis algorithm of Brentjens & de Bruyn (2005). The RM synthesis algorithm is described in detail in POSSUM Report #1.[1]

2. SOME IMPORTANT DETAILS

There are a few quantities that we should define before discussing RMCLEAN. The RM synthesis process takes as input a complex polarization spectrum $P(\lambda^2) = (Q(\lambda^2), U(\lambda^2))$. We choose values of Faraday depths ϕ_j at which we would like to sample the quantity known as the Faraday dispersion function (FDF):

$$\tilde{F}(\phi_j) = K \sum_{i=1}^{N} w_i P_i e^{-2i\phi_j(\lambda_i^2 - \lambda_0^2)} \,, \tag{1}$$

where the summation is over all λ^2 channels, w_i is the weighting at each channel, K is a normalization constant defined as

$$K = \left(\sum_{i=1}^{N} w_i \right)^{-1} \,, \tag{2}$$

and λ_0^2 is the weighted mean of the observed λ^2 such that

$$\lambda_0^2 = K \sum_{i=1}^{N} w_i \lambda_i^2 \,. \tag{3}$$

For a uniform weighting, $w_i = 1$ and $K = N^{-1}$ such that the weighted mean λ^2 reduces to just the straight mean of the λ^2 sampling. A crucial quantity for the RMCLEAN algorithm is known as the rotation measure spread function (RMSF) and is calculated from the λ^2 sampling of the observations and the weights at each channel:

$$R(\phi_j) = K \sum_{i=1}^{N} w_i e^{-2i\phi_j(\lambda_i^2 - \lambda_0^2)} \,. \tag{4}$$

This quantity informs our choice of Faraday depth sampling: we require the Faraday depth channel spacing to be much smaller than the FWHM of the RMSF, which for uniform weighting is approximately $2\sqrt{3}/\Delta\lambda^2$, where $\Delta\lambda^2 = \lambda_{max}^2 - \lambda_{min}^2$.

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[1]Our method for implementing RM synthesis has changed since Report #1: we now weight the polarization vector by the inverse square of the sensitivity rather than using a uniform weighting. This is discussed in detail in the current report.

3. KNOWN ALGORITHMS

We currently know of only three methods that allow one to reconstruct the Faraday depth components from a Faraday dispersion function:

1. **Standard Högbom Clean:** Heald (2009) presents the possibility that the rotation measure spread function (RMSF) can be used to deconvolve a dirty Faraday dispersion function via the CLEAN method developed by Högbom (1974) for use in aperture synthesis. The full details of this scheme are revealed in Appendix A of Heald et al. (2009). George Heald has developed a Fortran implementation of this algorithm for use in MIRIAD; this can be downloaded from his website at http://www.astron.nl/~heald/software.html. We have currently chosen to adopt this algorithm in our POSSUM pipeline, but have rewritten George's code in IDL and added some features. We discuss this RMCLEAN algorithm and our implementation in detail in the next section.

   We note that in the field of aperture synthesis Högbom's CLEAN was the earliest CLEAN algorithm and has been followed by a series of improvements including the Clark algorithm, the Cotton-Schwab algorithm, and the Maximum-Entropy Method (MEM) for deconvolution. To our knowledge, none of these improvements have been implemented in cleaning dirty Faraday dispersion functions.

2. **Wavelet-based RM Synthesis:** Frick et al. (2010) point out that the standard RM synthesis technique of Brentjens & de Bruyn (2005) fails to reproduce the phase and amplitude of the Faraday components that were used in the modelling of the latter's Appendix B. They point out the Brentjens & de Bruyn (2005) algorithm requires a Fourier transform over all λ^2 while observationally one measures polarization data only at $\lambda^2 > 0$. This leads to difficulties in trying to reconstruct the phase and amplitude of model components. Frick et al. (2010) describe the use of wavelets in a redesigned implementation of RM synthesis. While the technique looks promising, a detailed description of implementing the algorithm is not offered. Since the algorithm is still in development, we have not chosen to implement this scheme.

3. **Compressive Sampling-based RM Synthesis:** Li et al. (2011) have redesigned the RM synthesis algorithm using the advanced signal processing concept of "compressive sampling," a new scheme that IEEE Signal Processing Magazine (March 2008) describes as "a sampling paradigm that goes against the common knowledge in data acquisition." The examples from their paper show that this method is far superior to standard RMCLEAN (and even the wavelet-based method) in reconstructing the phase and magnitude of modelled Faraday depth components. The hearsay consensus among the POSSUM experts has suggested that this compressive sampling technique is computationally expensive (hence our focus on the first method above), but Li et al. (2011) claim their algorithm outperforms standard RMCLEAN by a factor of two. The algorithms are implemented in MATLAB and can be downloaded from a Google website listed in their paper. **WG8 really must investigate using the CS technique.**

We emphasize that only the first method above involves using the *exact* Brentjens & de Bruyn (2005) RM synthesis algorithm (which is what POSSUM has currently adopted). The latter two make adaptations to the algorithm outlined in §2.

4. THE RMCLEAN ALGORITHM

George Heald has produced a MIRIAD routine called RMCLEAN in Fortran. A description of this RMCLEAN algorithm is outlined by Heald et al. (2009):

1. In each spatial pixel, the complex $(Q(\phi), U(\phi))$ spectrum is cross-correlated with the complex RMSF. The location of the peak absolute value of the cross-correlation, ϕ_m, is noted.

2. If $P(\phi_m)$ is greater than a user-defined cutoff, a shifted and scaled version of the complex RMSF is subtracted from the complex $(Q(\phi), U(\phi))$ spectrum. The scaled RMSF is $gP(\phi_m)R(\phi - \phi_m)$, where g is a (real) gain factor.

3. The value $gP(\phi_m)$ is stored as a clean component.

4. Steps 1–3 are repeated until the value of $P(\phi_m)$ is no longer higher than the cutoff, or a maximum number of iterations have been performed.

5. Finally, the clean components are convolved with a restoring Gaussian beam with a FWHM equal to $2\sqrt{3}/\Delta\lambda^2$ and added to the residual $F(\phi)$. The result is the deconvolved $F(\phi)$ spectrum.

Notice that step (1) differs from the standard CLEAN algorithm in that a cross-correlation is made with the RMSF before finding a peak. Heald et al. (2009) explain:

> The reason for using a cross-correlation in step (1), rather than simply searching for the peak $P(\phi)$ (in analogy to imaging deconvolution), is the hope that incorrect component localization due to possible sidelobe confusion will be eliminated. In practice, the two techniques were found to yield the same results in our data.

Using George's MIRIAD code on both real and simulated data, we found extremely different results between the correlation method and the simpler peak method. USyd grad student Chris Hales also discovered this discrepancy and has pointed out a drawback of the algorithm that George has implemented. In step (2) above, one multiplies the gain factor by the shifted RMSF. Because the RMSF is shifted, one needs to calculate the RMSF out to a Faraday depth twice as large as the extent of the FDF. In the cross-correlation scheme (named "xcorr" in George's code), prior to correlation the RMSF is truncated to extend only to the maximum Faraday depth of the FDF and is then padded with zeros at absolute Faraday depths greater than this extent. We think the extended RMSF should be used in this scheme. In practice, we find this causes the cross-correlation scheme in George's RMCLEAN step (1) to be susceptible to cleaning low-amplitude peaks at extreme Faraday depths even when a brighter component at lower absolute Faraday depths needs cleaning. In light of these difficulties, we have decided to replace his step (1) with the simple method of finding the peak as is done in the Högbom aperture synthesis CLEAN.

Heald (2009) investigates whether or not to weight the Stokes Q and U data by the inverse of the square of the SNR when constructing the FDF and finds that weighting "provides a significantly better determination of" the Faraday depth at the peak of the FDF. We note that George's MIRIAD implementation of RMCLEAN does not allow for weights to be applied, but our IDL implementation of the algorithm does allow for weights to be passed via a keyword (with uniform weighting assumed as the default if the keyword

is not passed). POSSUM has decided to adopt the scheme of weighting the polarization data by the inverse of the band-averaged noise in Q or U. If the fractional polarizations, i.e., Q/I and U/I are input to RM synthesis, then the noise should be that of the fractional polarization in each channel, which may scale somewhat differently from the noise in Q and U because of spectral index effects in Stokes I.

Heald (2009) also points out that when weighting in the above fashion, "the RMSF is significantly broadened." Unfortunately, my current implementation of RMCLEAN assumes that the restoring function has a FWHM equivalent to the unweighted RMSF, namely $2\sqrt{3}/\Delta\lambda^2$. I believe this is not correct: we should be using a more accurate estimate of the FWHM of the weighted RMSF. **This absolutely needs to be investigated by WG8!**

The RMCLEAN algorithm deconvolves the RMSF from the dirty FDF. It is crucial to the success of the RMCLEAN algorithm that the RMSF be weighted in *exactly the same fashion* that the polarization data are when constructing the FDF. The FDF is rotated to λ_0^2 because this is where the variation in the imaginary part of the RMSF is minimized. Equation 3 is derived by setting the derivative of the RMSF with respect to the Faraday depth equal to zero. Hence, the weights w_i in eq. 3 are the same weights that appear in the calculation of the FDF and the RMSF shown in eqs. 1 and 4, respectively.

After conferring with Chris Hales and Bryan Gaensler, we determined that their implementations of RM synthesis involve rotating to the straight mean λ^2, rather than the weighted mean λ^2. This is always the case for George's RMCLEAN code because he hasn't allowed for weighting in his RMCLEAN. This means that even though Chris and Bryan's code the FDF and RMSF were calculated using weights, the cleaning wasn't being done at exactly the ideal place as derived by Brentjens & de Bruyn (2005). This isn't a problem as long as the RM synthesis produces a dirty FDF that is rotated to a given λ_0^2 and the RMSF that is used to clean said dirty FDF is also rotated to the same λ_0^2. Regardless, we should agree to follow the spirit of the RM synthesis algorithm and do the cleaning at the ideal λ_0^2 calculated using the weights.

Another concern is that of "derotation." The cleaning step is done after rotating to λ_0^2. As a final step, George's MIRIAD routine shifts each of the clean components back to the $\lambda = 0$ domain. In some of our tests, Chris Hales and I found that this caused high-frequency ripples to appear in the cleaned spectrum. We expect that the real and imaginary parts of the FDF will oscillate more across the extent of the main lobe of the RMSF when derotated back to $\lambda = 0$,[2] but the amplitude of the FDF should be the same in either domain. When implemented in IDL, we find the same exact behavior. We contacted George Heald about this in August 2010 but haven't heard back. Robert Braun recently pointed out that this behavior is very likely due to inadequate sampling in Faraday depth. This should be relatively easy to characterize, but has yet to be done.

Do we want to derotate cleaned FDFs back to $\lambda = 0$ or leave them in the weighted mean λ^2 frame, λ_0^2, where the cleaning is done? If we derotate back to $\lambda = 0$ then we can properly measure polarization angles. If we leave the FDF in the λ_0^2 domain, then according to Brentjens & de Bruyn (2005), we "can still properly analyze spatial coherence of polarization angles in a spatially extended source at a certain Faraday depth."

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[2]Compare figures 3 and 4 in Brentjens & de Bruyn (2005).

As a final note, George's MIRIAD code defaults to returning the cleaned FDF derotated from the λ_0^2 frame back to $\lambda = 0$, but the code passes back the model clean components and residuals in the λ_0^2 domain; if derotation is being applied, we believe that it should be applied to all returned products.

5. IDL IMPLEMENTATION OF RMCLEAN

While trying to run the MIRIAD RMCLEAN code on both the ASKAP simulations and large ATCA data cubes, I found the code to be extremely slow. I therefore implemented the algorithm in IDL keeping performance for large data cubes in mind. A standalone IDL program named RMCLEAN.PRO is now available and will be offered to the public via my website. The final implementation of the algorithm in the ASKAP/POSSUM pipeline will be designed by the ASKAP computing team.

6. SUMMARY

The POSSUM team has decided to implement the scheme of performing RM synthesis as described by Brentjens & de Bruyn (2005) followed by a deconvolution of the RMSF using the RMCLEAN algorithm of Heald et al. (2009). We have successfully implemented these on the first round of ASKAP simulations using efficient and well-documented IDL code (for results, see POSSUM Reports 4 and 16). The ASKAP computing group will implement the algorithm described here in their own ASKAPSoft package.

Our investigations into the RMCLEAN algorithm have brought about the following action items that need to be followed up by POSSUM WG8:

- When using non-uniform weighting, a reasonable estimate for the FWHM of the weighted RMSF should be made so that our restoring function is accurately reflecting the underlying RMSF. The standard estimate of $2\sqrt{3}/\Delta\lambda^2$ only holds for uniform weighting.

- We need to decide whether our cleaned spectra should be derotated to $\lambda = 0$ or left in the λ_0^2 domain.

- If derotating to $\lambda = 0$ is preferred, we should investigate what minimum Faraday depth sampling is needed in order to allow for accurate reconstruction of the FDF.

- Given the benchmark results of Li et al. (2011),[3] our recommendation is that the compressive sampling technique be applied to POSSUM test data in order to determine whether it is in fact as computationally expensive as has been claimed. Since the code is readily available, it should be tested as soon as possible before the RMCLEAN algorithm is locked into the ASKAP pipeline.

REFERENCES

Brentjens, M. A., & de Bruyn, A. G. 2005, A&A, 441, 1217
Frick, P., et al. 2010, MNRAS, 401, L24
Heald, G. 2009, in IAU Symposium 259, IAU Symposium, 591
Heald, G., et al. 2009, A&A, 503, 409
Högbom, J. A. 1974, A&AS, 15, 417
Li, F., et al. 2011, A&A, 531, A126+

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[3]Larry Rudnick has recently reported the discovery of "pseudo-canals" in EVLA data that are brought about via interference of RM components. After consultation with Michiel Brentjens, he has suggested that the compressive sampling algorithm would be better suited to the analysis of such data, but also repeats the claim that the CS algorithm is computationally expensive compared with RMCLEAN.