PROBLEM SET 10, ASTRONOMY 216

Due Tuesday, 17 April 2001

1. a. You observe a spherically symmetric expanding shell of H I in the 21-cm line (or some other optically thin emission line) using a telescope whose beamwidth is much broader than the diameter of the shell. Thus, you obtain a single spectrum of brightness temperature T_B versus velocity V. Derive the shape of this spectrum, assuming that the expansion velocity V_{ex} is very much greater than the velocity dispersion of the gas within the shell ΔV, and that the 21-cm line is optically thin. Do you see limb-brightening?

b. Next, you observe the same object but with higher angular resolution—enough resolution to resolve the details of the shell structure. However, you don't have any velocity resolution, so you see the integrated line intensity without any kinematic details. Determine the limb-brightening for the velocity-integrated spectrum (which is equivalently, of course, the limb-brightnening for N_{HI}). Here, we define "limb-brightening" as the ratio of brightness at the edge, where the path length through the shell is longest, divided by the brightness at the center of the shell.

c. Now consider the case when you have enough velocity resolution to easily resolve the details of the spectrum. Suppose that the shell has volume density n, radius R, and thickness ΔR, that the velocity gradient within the shell is zero, and that $\Delta V/V_{ex} < (\Delta R/R)^{1/2}$; work to first order in $\Delta V/V_{ex}$ and $\Delta R/R$.

Determine the limb-brightnening for the *peak* brightness temperature in the spectrum (as opposed to the *velocity-integrated* brightness temperature). Does the peak brightness temperature exhibit limb-brightening? (The answer: it depends...).

2. The upper attached diagram is a contour map of brightness temperature of the 21-cm line vs galactic latitude and radial velocity for galactic longitude $\ell = 45°$ (taken from the Weaver and Williams 21-cm line survey of the Galactic plane, Astron. Ap. Suppl. vol 17).

a. What radio astronomers really measure is brightness temperature as a function of radial velocity (i.e., frequency) at a particular position; this map was synthesized from many such measurements. Such a measurement at a particular position is called a 'line profile'. Sketch the line profiles for $(\ell, b) = (45°, 0°)$.

b. For $\ell = 45°$, write down *all galactic radii* and *all distances from the sun* for the H I emitting at observed velocities of −50, 0, and +50 km/sec. Remember that positive velocities mean motion away from the observer. Assume that all motions are a result of Galactic rotation; that the Galactic rotation velocity is 220 km/s, independent of radius; and that the Sun's distance from the Galactic center is 8.5 kpc. *Hint:* Burton (1988, in *Galactic and Extragalactic Radio Astronomy, 2nd Edition*, ed. G.L. Verschuur and K.I. Kellermann, p. 295) discusses the Galactic rotation curve, observed velocities, and the derivation of distances therefrom.

Note that the negative-velocity gas is displaced from $b = 0°$; this reflects the 'Galactic warp', a bending of the Galactic plane at large Galactocentric radii.

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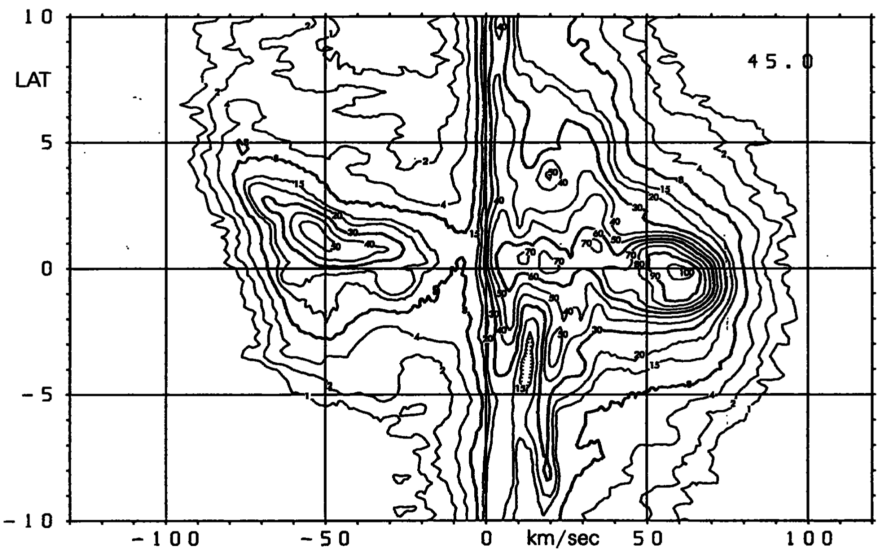

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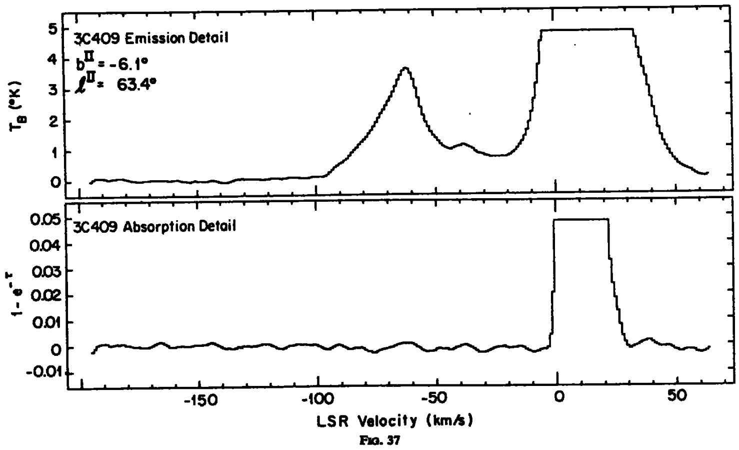

FIG. 37