PROBLEM SET 12, ASTRONOMY 216
FARADAY ROTATION/DEPOLARIZATION

1. This problem illustrates Faraday depolarization in two idealized cases. First, consider a uniform medium in which relativistic electrons generate synchrotron emission, which is $\approx 70\%$ linearly polarized, and thermal electrons depolarize the emission. The medium extends for length L along the line of sight, and the total Faraday rotation angle along L amounts to θ_0 (which varies as λ^2, of course).

a. Before beginning the meat of this problem, we will first define a few things about Stokes parameters. Stokes parameters are useful because the individual contributions to any one parameter add up to make the total and you don't have to worry about combining things at different angles. Stokes parameters are defined in Rybicki and Lightman, pages 62-69.

The state of polarization is normally calculated from measurements of the apparent intensity by equipment sensitive to two orthogonal states of polarization, denoted say by a and b. For example, to measure the state of linear polarization, one makes a measurement with equipment sensitive to E-vectors oriented at angles 0° and 90°, and then a second measurement with the equipment rotated so that the sensitivity is to angles of 45° and 135°.

For the 0°-90° measurement, show that the *difference* between the between the two polarizations yields Stokes parameter Q and the *sum* yields Stokes parameter I. For the 90°-135° measurement, show that the difference yields Stokes parameter U and the sum yields I.

Note that the implication here is that two separate measurements of the orthogonal polarizations, at different angles, are required to completely specify the linear polarization: a linearly polarized wave at angle 45° has Q=0, so it will look unpolarized if you happen to observe at an unfortunately-chosen angle.

b. This leaves Stokes parameter V. If you choose the orthogonal polarizations properly, then the difference between the two polarizations yields V directly (and the sum again yields I). What is the proper choice? What is the approximate value of V for our synchrotron-emitting region?

c. For our region, determine the angle of apparent polarization (*i.e.*, the angle measured by the observer) as a function of θ_0. Then determine the apparent degree of polarization as a function of θ_0.

d. Now, to illustrate the effects of fluctuations in the field, consider a second case. Suppose the region of length L abuts a second region of length L, precisely similar in all respects except that the field is pointed in the opposite direction. Repeat problem *c*. Now consider: if you were an observer, and had nothing but these data, would you be able to distinguish between *c* and *d*?

Note: There is a reasonably extensive literature on the depolarization effects arising from Faraday rotation for regions as diverse as quasar cores, jets, and the interstellar medium. In the ISM, a recent spate of observational papers is providing new info about the magnetic field strength

and small-scale structure; the best example is Gaensler et al (2001 ApJ, 549, 959). For theoretical developments, the classic papers are Burn (1966 MNRAS, 133, 67) and Cioffi and Jones (1980 AJ, 85, 368).