PROBLEM SET 2, ASTRONOMY 216

1. Estimate the approximate value of the optical depth at the middle of the Lyα line from the center to edge of an ionization-bounded H II region, $\tau_{L\alpha}^{edge}$. Take the H II temperature as 10^4 K, the volume density as 10^3 H-nuclei cm^{-3}, and assume an O5 star. Also assume that the H II region contains no dust. The point is to show that it is typically very large, not to get an exact value; thus make reasonable approximations. To accomplish this feat:

a. First, calculate the Ly-*continuum* optical depth averaged over ionizing photon energy from the center to edge of the H II region, τ_{Lc}^{edge}. Define the edge as being the point where $n_e^{edge} = n_{HI}^{edge}$, i.e. where the ionization fraction $x_e = 1$. *Hint:* It is very difficult to obtain an exact expression for the edge. However, you can obtain an accurate result by making shrewd approximations. You may assume that the edge is located at the Stromgren radius, and neglect electrons contributed by He. Most of the HI is located right near the edge, so that is where you should put most of your attention.

It is possible to express τ_{Lc}^{edge} in terms of only the ratio $\langle n_e^{HII}/n_{HI}^{HII}\rangle$, where the superscript HII denotes values within the H II region and the averaging is done over the H II region; you can then estimate an approximate value for this ratio. If you do this, you will find τ_{Lc}^{edge} depends only very insensitively on the volume density, type of exciting star, or any other properties of the H II region. Can you justify this phenomenologically in a simple way?

b. Next, calculate the ratio of Lyα optical depth to the Ly-continuum optical depth, $\tau_{L\alpha}/\tau_{Lc}$. Note that the continuum optical depth is an integral of the product of the cross section over frequency; use any reasonable approximation for this integral. To make things easier we suggest you assume that the Lyα line profile is rectangular with an appropriate thermal width (namely, the full width at half maximum, or FWHM; Spitzer, section 3.2a). To do this problem you need to know the transition probability for Lyα which can be found in Rybicki and Lightman page 280; pay attention to using the correct value of g. Spitzer, chapter 3.2, will also help.

2. This problem illustrates the quandary produced by "ultracompact H II (UC H II) regions". They are small, high-density H II regions that were discovered long ago in specially-chosen places, namely some of the major H II regions of the Galaxy, but Wood and Churchwell (1989, ApJSuppl, 69, 831; WC) made the first approximation to a large-scale census (using statistical sampling, not the complete count so highly favored by some politicians). The INTRODUCTION to their paper contains an excellent, concise description of why these regions are important, which involves fundamental issues of the formation rate of massive stars, the environments in which they form, the gasdynamical puzzle of age, and the Galactic distribution; **YOU SHOULD READ THEIR INTRODUCTION,** and preferably the body of the paper (which will help you do this problem). A recent review by Kurtz ete al (2000, in Protostars and Planets IV, page 299) brings you up to date on observational and theoretical aspects, but the essentials have not evolved much. García-Segura and Franco (1996, ApJ 469, 171) provide a theoretically-oriented discussion, and also include the development of instabilities ("elephant trunks") at the edge that look like the famous Hubble

picture of Orion.

a. Look at G15.04–0.68 in Table 14; also read their blurb about it in §IVc. This is located in the famous H II region M17 (M means it's in Messier's catalog, meaning that it is bright enough to have made it!). With the VLA parameters used for the survey, WC didn't have enough angular resolution to see its internal structure so they classify it as a spherical H II region; in fact, it is a shell (see Felli, Churchwell, Massi 1984 AAp 136, 53; FCM; this paper also gives some nice maps of the larger-scale region in which the UC H II region is embedded). But for the present, consider it spherical. FCM cite data showing that the turnover frequency, where the H II region becomes optically thick, is about 22 GHz.

Determine the approximate emission measure, diameter, electron density, ionizing photon luminosity of the central star (and, from that, spectral type of the central star). You can check your results from WC Table 16.

b. The VLA is an array of telescopes, each pair of which acts as a standard Michaelson interferometer having a fringe spacing on the sky given by the well-known formula of Freshman Physics. The VLA attains high angular resolution because with what is known as "aperture synthesis techniques" one can map a source with an angular resolution that is equal to the fringe spacing. For an observing frequency of 22 GHz, the highest frequency usable at the VLA, what antenna separation is required to map the source with ten resolution elements across its diameter (i.e., fringe spacing equal to 1/10 of the H II region diameter)? (A proposal for an upgraded VLA, one that would provide this and more, has recently been submitted).

c. Suppose that the H II region is imbedded within a cold, neutral cloud having the same volume density that exists within the H II region itself. Under these conditions the H II region will expand approximately according to Spitzer equation 12-20, except that the factor $\frac{7}{4}$ within the parenthesis should be replaced by 2.02 (as we might derive in class next week). WC's rather arbitrary, but practical, definition of being a UC H II region, instead of just a run-of-the-mill H II region, is that its angular diameter < 10 arcsec. Approximately, how long will the region to expand to this size? What does this say about the probability of finding UC H II regions? What's the electron density in the ionized region at this epoch (it's *not* down by the ratio of the ionized volumes!)?

Note: the short lifetimes of UC H II regions means that few should be in existence at any one time; WC elaborate on this point in §IVb and mention two possibilities for reconciliation with the much larger number that is observed. Note, also, that many UC H II regions have shell morphologies; in §IVa WC consider the question, "What keeps the gas from filling in the shell's interior?" (which would happen rapidly, typically ~ 300 yr) and suggest a stellar wind.