PROBLEM SET 5, ASTRONOMY 216

1. This problem concerns moving stars and H II regions. Consider an O5 star moving through a uniform medium of H-atom density n_0 at a velocity $v_0 = v_*$. The motion of the star disturbs the H II region. Assume that the ionization front and the flow are steady, that the flow is one-dimensional and parallel to the star's motion, and that the ionized gas velocity v_{II} and density n_{II} within the H II region are constant (i.e., independent of position within the H II region). This problem was treated by Raga (*Ap. J.* **300**, 745); here, just determine R, the distance to the 'nose' of the H II region, the point towards which the star is moving.

a. First, set up the ionization equilibrium for the 'nose' in terms of Raga's β and $\eta = \frac{R}{R_{SII}}$. Note that Raga assumed $v_{II} \gg 2c_{II}$ and defined β in terms of v_* and n_0; here, be more general and define β in terms of v_{II} and n_{II}, so that $\beta = 4\pi n_{II} v_{II} R_{S_{II}}^2 / N_u$, where $R_{S_{II}}$ is the usual Strömgren radius defined with an electron density n_{II} (as oppposed to R_{S_0}, which is defined for an electron density n_0).

You should obtain a cubic equation to solve. Although such an equation may be solved exactly, the solution is not very illuminating. Instead, you should develop the simplest of approximations to the solution by considering the limits $\beta \to 0$ and $\beta \gg 1$, and connecting the two in a reasonable way. You should be able to obtain a simple relation that is accurate to better than 20%. Determine the approximate accuracy of your approximation.

b. Determine the distance to the 'nose' for an R-type ionization front and $v_* \gg 2c_{II}$ in terms of useful parameters (v_{II}, n_{II}, and $R_{S_{II}}$ are not 'useful' because you have to calculate them, while v_*, n_0, and R_{S_0} are 'useful' because they are the 'given parameters' of the problem. You can regard c_{II} as 'useful' because it is just obtained from the standard temperature of H II regions.) *Hint:* in this case, what are n_{II}/n_0 and v_{II}/v_0?

Evaluate the distance in terms of R_{S_0} for $v_* = 100$ km s^{-1} and $n_0 = 100$ H-atoms cm^{-3}. Neglect Helium.

c. Do the same for a D-type ionization front, for $2c_{II} > v_* > c_0$. Assume the ionization front is D-critical and that the shock is strong and isothermal. In this case, matter can accumulate in the ionization front, so $n_{II} v_{II} \neq n_0 v_*$. Again, express the results in terms of useful parameters.

Evaluate the distance in terms of R_{S_0} for $v_* = 3$ km s^{-1} and $n_0 = 100$ H-atoms cm^{-3}. Take $T_{II} = 7000$ K, $T_0 = 70$ K, and neglect Helium.

2. 'Champagne flow' or 'blister' HII regions: Many H II regions are observed to be at or near the edge of the large molecular cloud from which the ionizing star was born. Consider a star that is initially located at the boundary of a semi-infinite cloud. After the star turns on, the R-type ionization front will create a hemispherical region of ionization in the cloud; assume that the H II region remains hemispherical after the D-type front becomes established and the H II region begins expanding. Neglect considerations related to conversion between H_2 and H I in the shock.

a. Using an argument similar to that given by Spitzer for spherical H II regions, find an approximate expression for the radius of the shock front as a function of time. Assume that the ionized gas can expand freely into the medium outside the cloud (which means that you can assume that the ionization front is D-critical).

Your result for $R(t)$ should not differ much from Spitzer's.

b. Make an approximate calculation of the mass of gas that has been ionized and has left the H II region at a time $t \gg (R_{st}/C_{II})$; R_{st} is the Stromgren radius for a density equal to the cloud's density. Compare this mass with that piled up at the edge of the hemispherical shell.

You can assume that as the gas crosses the hemispherical boundary that separates the cloud from the ambient medium, it has velocity C_{II}; this is consistent with the D-critical assumption.