THE CO MOLECULE

1. The CO molecule has its first rotational transition ($J = 1 \rightarrow 0$) at a frequency of 115 GHz.

a. Write an expression for the energy W_J of rotational level J for CO. Express the energy in units of temperature. Also, write the degeneracy of each J-level.

b. Suppose you have a total volume density $n(CO)$. Assume that the volume density is so high that the level populations are collisionally dominated. Write an expression for the number $n(J)$ of molecules in each level J. Make sure you include the proper statistical weight factors for the levels. *Also*, make sure that your sum over $n(J)$ gives the total number $n(CO)$—*i.e.*, make sure that you include the partition function, and properly normalize your distribution of $n(J)$.

c. Calculate numerical values for $\frac{n(J)}{n(CO)}$, for $J = 0, 1$ for $T = 30$ K and $T = 5$ K. In the optically thin limit, the line intensity $\propto$ the number in the upper state. An observer who observes the 115 GHz (or any other line, for that matter) line has to know the temperature if she wants to determine the *total* CO abundance. The CO line is rarely optically thin. However, its less-common isotopes such as ^{13}CO are commonly optically thin and used to determine CO abundance.

d. For $kT \gg W_{(J=1)}$, the partition function can be approximated as an integral. Express the answer to (b) as a simple function of temperature, by evaluating the integral.

e. The optical depth of any line $\propto N_1(1 - \frac{n_2 g_1}{n_1 g_2})$, where the subscripts 1 and 2 indicate the lower and upper states. For CO, how does the optical depth of the ($J = 1 \rightarrow 0$) line depend on temperature in this high-temperature limit, assuming LTE?

How does the optical depth of a two-level system such as the CII FS IR line depend on temperature? Explain in phenomological terms why these temperature dependencies differ.

f. Determine the critical density for the levels $J = 0$ to 3 for temperatures 5 and 20 K. You will need collisional excitation rates for excitation of CO by H_2. You can find these in Table 7 of Green and Thaddeus (1976 ApJ, 205, 766). When you look these up, read the short ($\sim$ 1 minute!) easy section IIIg which describes the complications in molecule-molecule collisions and the even shorter section IVc which describes what approximations they actually made in their calculations. If you find a more modern reference, so much the better, but do read these short sections anyway—they are very instructive.

Note: Spitzer Table 4.2 also gives excitation rates for CO. However, they differ from Green and Thaddeus by a factor of 10!

Comment: We have assumed collisional domination. But notice how rapidly the critical densities rise with J. This means that the high-J levels may not be, in fact, collisionally dominated, meaning that their populations are smaller. In this case the populations also depend on the radiation intensity in the relevant line—which means they depend on the optical depths which, in turn, depend on the populations. So this case is not so easy!