Astro 250 – Planetary Dynamics – Final Project

1 Using a Symplectic Integrator to Explore Secular Oscillations

Write your own numerical symplectic integrator using the Wisdom-Holman (1991, AJ, 102, 1528) algorithm. Although the Wisdom-Holman algorithm is designed to treat $N \geq 2$ massive bodies and an arbitrary number of test particles, for this final project you need only write an integrator to solve the restricted (but possibly *non*-circular and *non*-coplanar) three-body problem: 1 central mass (the Sun), 1 massive perturber, and 1 test particle.

Work in units where the gravitational constant and the central mass are such that $GM = 1$. Unless indicated otherwise, take the perturber to have a mass $m_{\rm pert} = 10^{-3}$ of the central mass and to have an osculating semimajor axis of $a_{\rm pert} = 1$. At $t = 0$, the perturber is located on the negative x-axis, while the test particle is located on the positive x-axis.

Recall that the Wisdom-Holman algorithm is not designed for "close encounters" which occur when bodies approach one another within a few mutual Hill sphere radii. Watch for such close encounters and stop the integration if one occurs (or flag the integration and continue it for fun).

(a) *How constant is your Jacobi constant?* Test your integrator by seeing how well it conserves the Jacobi constant of the test particle, when the perturber occupies a perfectly circular orbit ($e_{\rm pert} = 0$).

At $t = 0$, take the test particle to have the following osculating elements: $a_0 = 0.1$, $e_0 = 0$, and $i_0 = 15$ degrees (all inclinations are measured relative to the perturber's orbital plane). Write down the initial conditions $(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)$ for the test particle. Do the same for the perturber (subscript "pert").

Plot the test particle's Jacobi constant $C_{\rm J}(t)$ from $t = 0$ to $t = 10^5$. A prize may be awarded for whomever best conserves the Jacobi constant.

(b) *Circular external perturber:* Having satisfied yourself that you have a decent numerical integrator, calculate the test particle's dynamical evolution for a range of initial inclinations $i_0 = 0$, 15, 30, 45, 60, 75 and 90 degrees. Keep $\mu = 10^{-3}$, $a_{\rm pert} = 1.0$, $e_{\rm pert} = 0$, and the initial $a_0 = 0.1$ and $e_0 = 0$. For each i_0, calculate $e(t)$, $i(t)$, $\tilde{\omega}(t)$, and $J_z(t)$ (the component of the test particle's angular momentum perpendicular to the orbital plane of the perturber) from $t = 0$ to $t = 10^5$.

(c) *Circular internal perturber:* Repeat (b), but for $a_0 = 3.0$ (test particle is exterior to perturber).

(d) *Dependence of precession rate on perturber mass:* For each of the following four cases $(a_0, i_0) = (0.1, 15\,\text{deg}), (0.1, 75\,\text{deg}), (3.0, 15\,\text{deg}), (3.0, 75\,\text{deg})$, calculate numerically how the precession period $2\pi/\dot{\tilde{\omega}}$ scales with μ, for μ between 0.1 and 10^{-6}. Compare with the analytic expectation.

(e) *Eccentric external perturber:* Repeat (b), but for $a_{\rm pert} = 10$, $e_{\rm pert} = 0.75$, $\tilde{\omega}_{\rm pert} = 0$, and $a_0 = 0.5$.

(f) *Eccentric internal perturber:* Repeat (b), but for $a_{\rm pert} = 1$, $e_{\rm pert} = 0.75$, $\tilde{\omega}_{\rm pert} = 0$, and $a_0 = 5.0$.