Astro 201 – Radiative Processes – Problem Set 3

Due in class.

Readings: Rybicki & Lightman 8.1, Mihalas pages 81–84 (his pages, not the Course Reader's), Mihalas section 4-5. Also however much of Rybicki & Lightman pages 90–102 that you wish.

Problem 1. Eddington Limit

Rybicki and Lightman problem 1.4

Problem 2. Pulsar Dispersion Measure

A radio astronomer notes that pulses from a certain pulsar observed at a radio frequency of $\nu = 2$ GHz arrive slightly ahead of the pulse train observed at $\nu = 1$ GHz. The lead time is 1 s.

(a) Use the dispersion relation derived in class for a cold, ionized plasma, and the information above, to derive the column density of electrons along the line of sight to this pulsar. Express in standard pulsar-community units of cm^{-3} pc.

(b) For an assumed density of electrons in the interstellar medium of $0.03\,\mathrm{cm}^{-3}$, calculate the distance to this pulsar. Does this seem reasonable?

(c) For such an assumed electron density, is the radio astronomer safely observing above the plasma cut-off frequency?

(d) Calculate the optical depth to Thomson scattering along the line-of-sight to this pulsar.

OPTIONAL Problem 3. Hyperfine $^3\mathrm{He}^+$

NOTE: While this problem is optional, it concerns material covered in lecture, and all material covered in lecture will be tested on the mid-term and final exams.

Observations of the hyperfine transition in $^3\mathrm{He}^+$ are used to probe the $^3\mathrm{He/H}$ abundance in the galaxy. This abundance reflects the primordial yield from big bang nucleosynthesis and galactic chemical evolution.

(a) Estimate, using the scaling relations presented in class and whatever facts you remember, the wavelength of the ground-state hyperfine transition in $^3\mathrm{He}^+$. Compare to the true answer of 3.46 cm.

(b) Estimate the Einstein A coefficient (transition probability) of this line. Compare to the true answer of 1.95×10^{-12} s^{-1}.

OPTIONAL Problem 4. Shades of the Sun

(a) When the sun sits 5 degrees above the horizon and is about to set, it looks red. Give a semi-quantitative explanation why. Neglect dust.

(b) During a total solar eclipse, the sun's corona appears white to the naked eye. The corona is composed of plasma. State why the corona looks white (one sentence will suffice).

Problem 5. Back-of-the-envelope CO 1-0

Most astronomers know that the Einstein A coefficient for the Lyman alpha ($n = 2$ to $n = 1$) transition in atomic hydrogen is of order $10^9\,\mathrm{s}^{-1}$ (actually, $5 \times 10^8\,\mathrm{s}^{-1}$) We estimated this result in class to order-of-magnitude by considering an (accelerating) electron on a spring that displaces a Bohr radius, a_0, and has natural angular frequency ω. We took the inverse lifetime of the excited state as $A \sim P/\hbar\omega$, where P is the power radiated by an accelerating charge.

The ubiquitous carbon monoxide molecule, ^{12}CO, is used by astronomers to trace the presence and temperature of molecular gas in everything from galaxies to circumstellar disks. We would rather try to detect H_2, but sadly H_2 has no permanent electric dipole moment because of its symmetry. Carbon monoxide does have a permanent dipole moment.

(a) Estimate the wavelength of the lowest energy, rotational transition in CO ($J = 1$ to $J = 0$, where J is the rotational quantum number). Do this by considering a barbell spinning about its axis of greatest moment of inertia and recognizing that angular momentum comes quantized in units of $\hbar$.

Compare your estimate to the true answer of 2.6 mm.

(b) Use the scalings of our semi-classical spring model to estimate the Einstein A coefficient of this transition, i.e., the inverse lifetime of the excited $J = 1$ state.

Bring to bear one fact that is difficult to guess from first principles: the dipole moment of the CO molecule is 0.1 Debyes (1 Debye = 10^{-18} cgs. Note that $ea_0 = 2.5$ Debyes, where e is the electron charge), not $\sim$1 Debye, as one might have guessed naively. The smaller-than-usual dipole moment of CO is a consequence of the strong double bond connecting C to O. Most other molecules—e.g., H_2O, CS, SiS, SiO, HCN, OCS, HC_3N—have dipole moments that are all of order 1 Debye.

Compare your estimate to the true answer of $6 \times 10^{-8}\,\mathrm{s}^{-1}$.

(c) Estimate σ, the cross-section of the transition at line center, in cm^2. Assume the line to be thermally broadened at a typical molecular cloud temperature of 20 K.

(d) If the number abundance of CO molecules to H_2 molecules is $n_{\mathrm{CO}}/n_{\mathrm{H}_2} \sim 10^{-4}$ (i.e., close to the solar abundance ratio of $n_{\mathrm{C}}/n_{\mathrm{H}} \sim 10^{-4}$), estimate the column of hydrogen molecules required to produce optical depth unity at the center of the CO line. Is this column likely to be exceeded in a molecular cloud? Take a typical molecular cloud H_2 density of $10^4\,\mathrm{cm}^{-3}$ and a cloud dimension of 100 pc.[1]

(e) Based on your answer to (d), would you conclude that this transition is a good way to measure the mass of molecular gas in a galaxy? What about the temperature?

[1] In Spitzer's book on the interstellar medium, molecular cloud densities of molecular hydrogen are said to range from 10^3 to $10^6\ \mathrm{cm}^{-3}$. The density can be orders of magnitude higher than even $10^6\,\mathrm{cm}^{-3}$ as you approach star forming regions. Molecular clouds are clumpy structures.