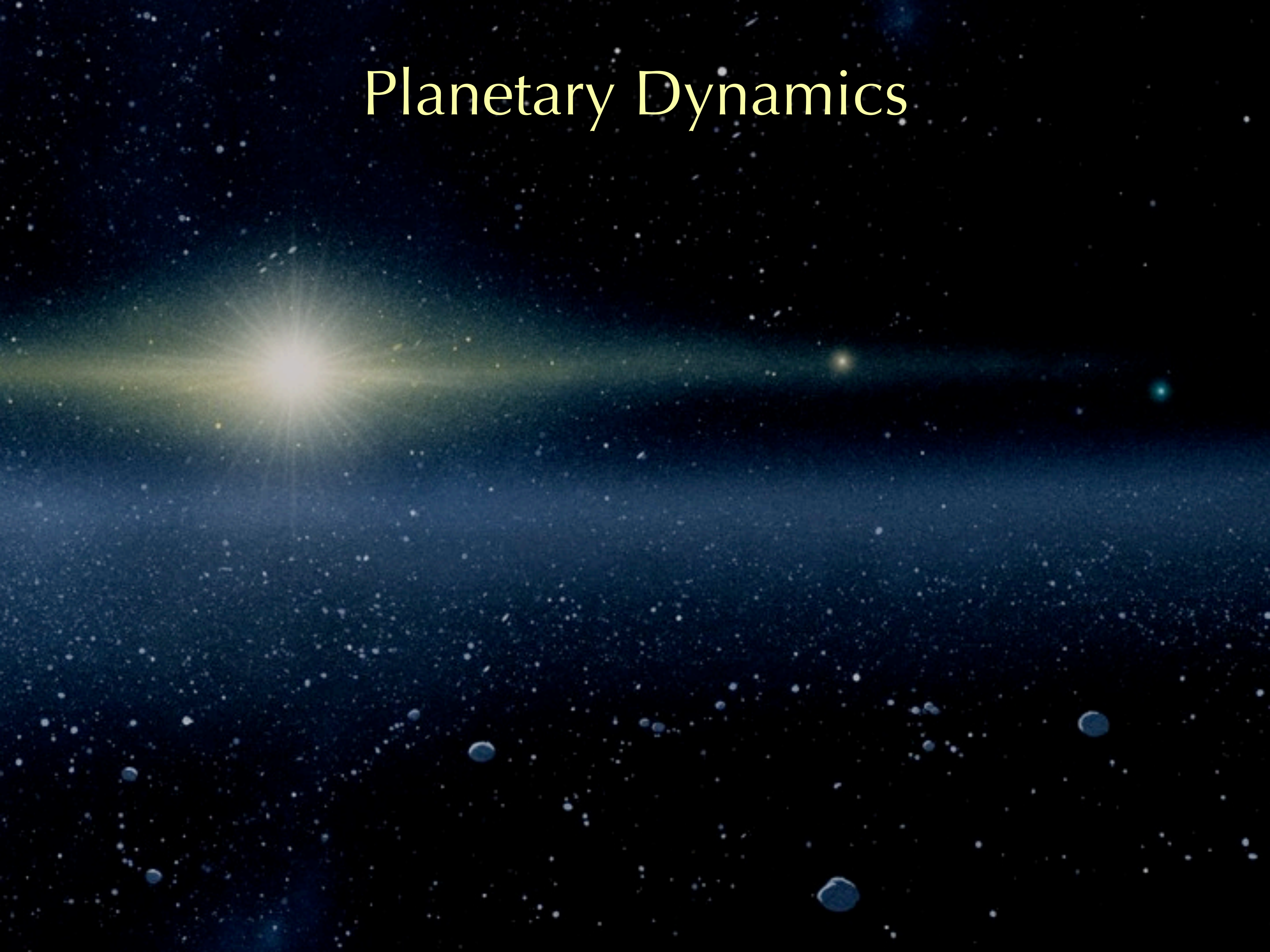
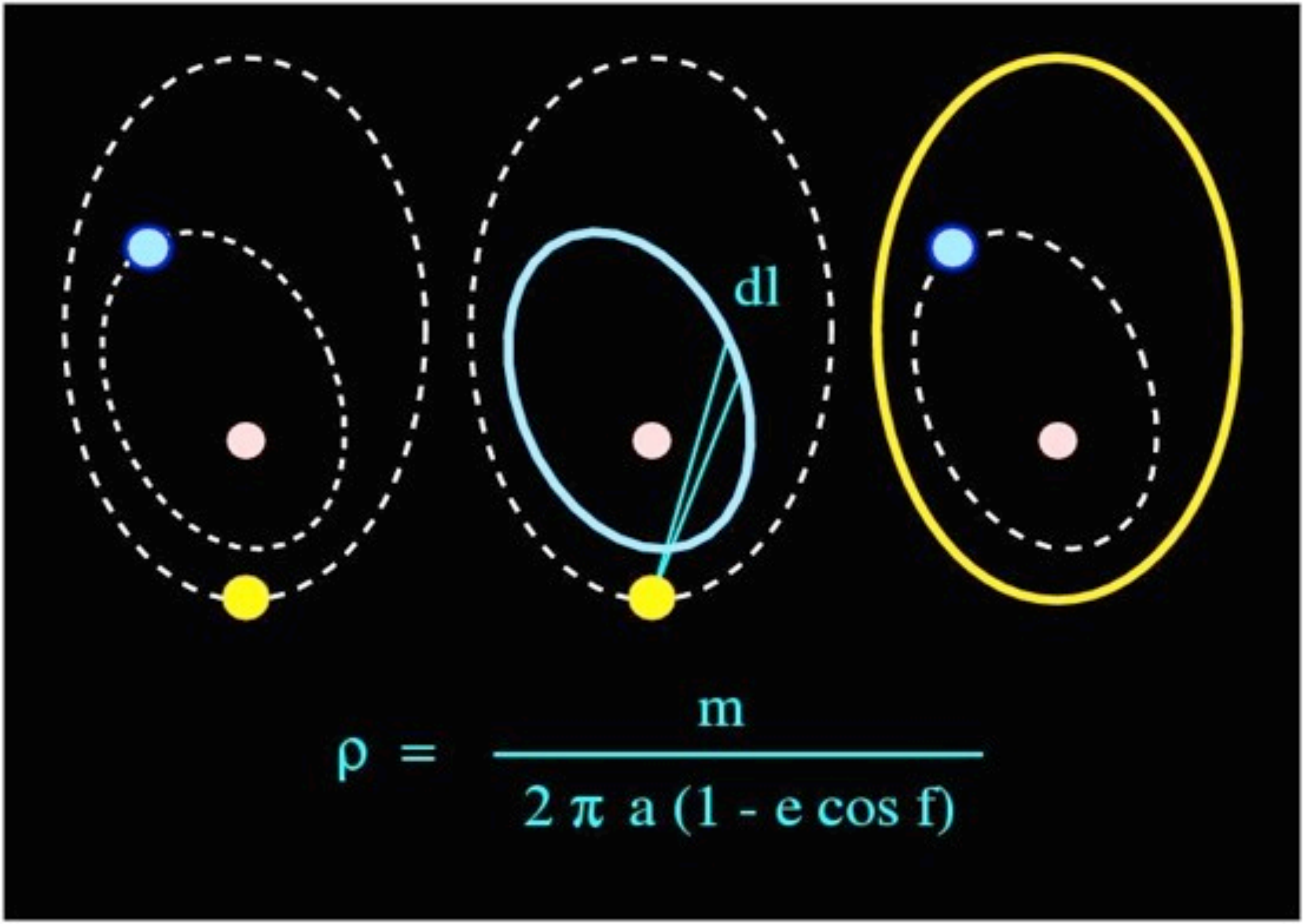
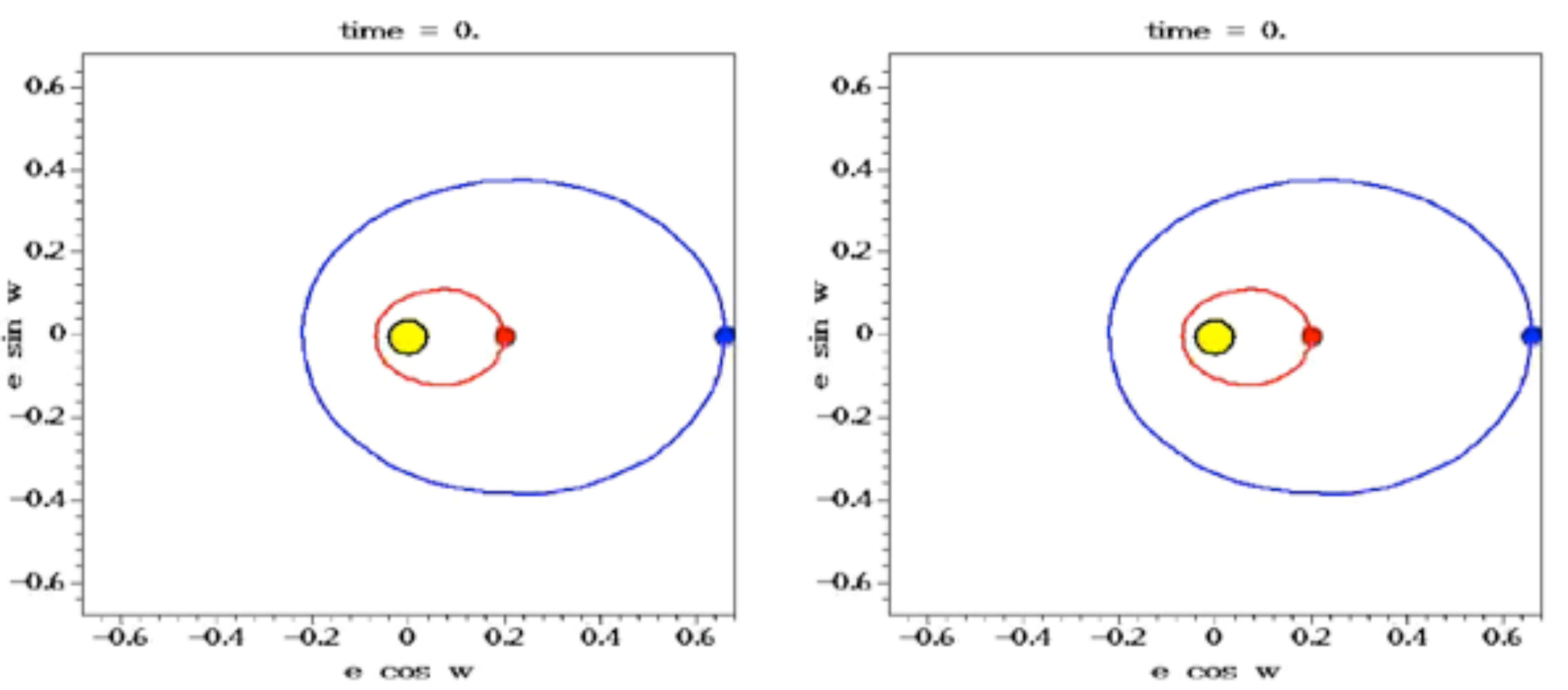
Planetary Dynamics



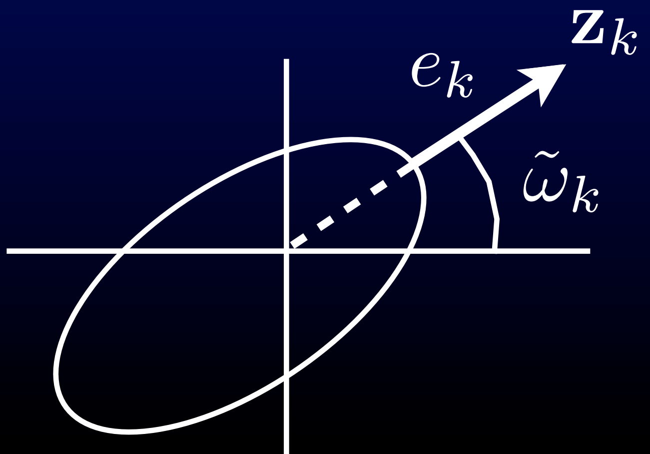
Secular ("long-term") dynamics: Replacing point masses with wires (longitude-averaged evolution)



$$\rho = \frac{m}{2\pi\, a\,(1 - e\cos f)}$$



Linear secular theory (keeping quadratic terms in Hamiltonian):
N planets ⇒ N eccentricity modes

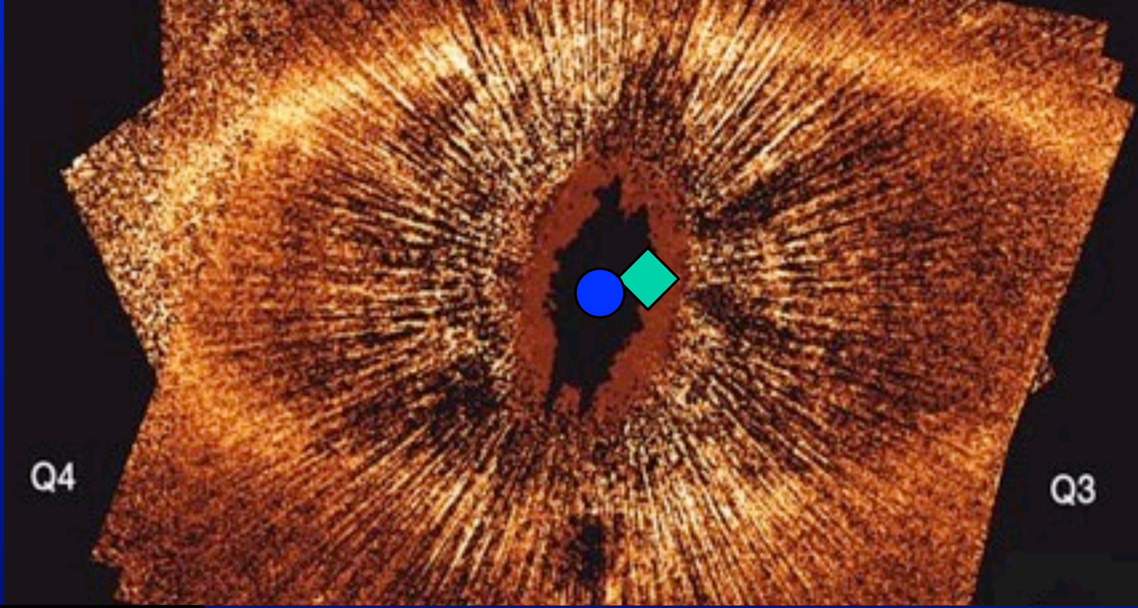


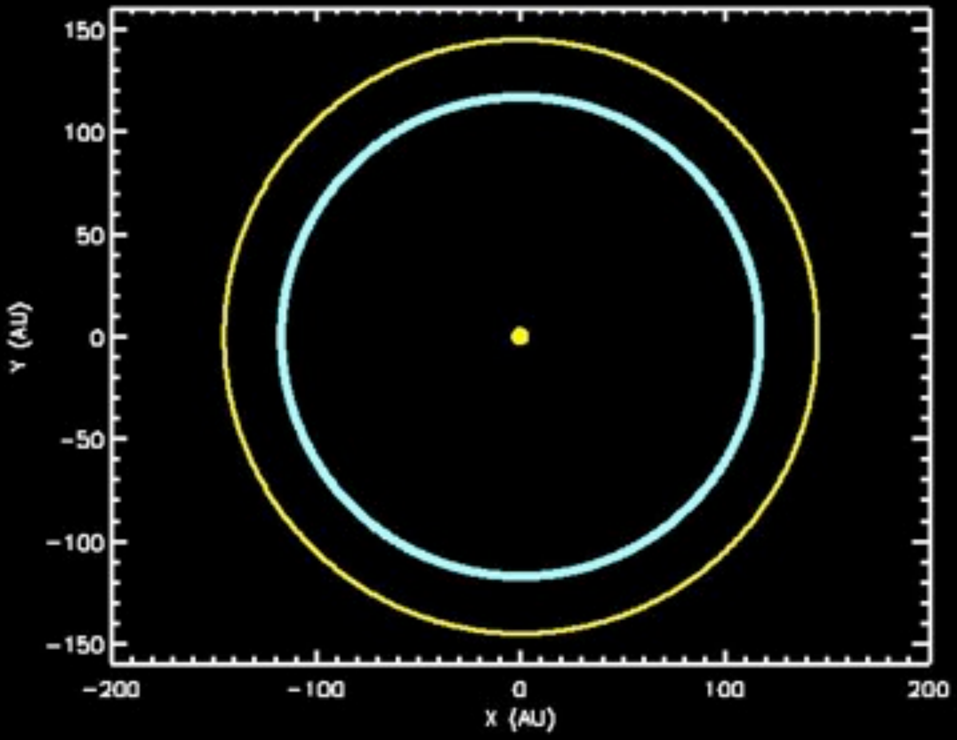
$$\tilde{\mathbf{z}}_{k\alpha} = e_{k\alpha} \exp[i(g_\alpha t + \beta_\alpha)]$$

$$\mathbf{z}_k = e_k \exp(i\tilde{\omega}_k) = \sum_{\alpha=1}^{N} A_\alpha \tilde{\mathbf{z}}_{k\alpha}$$

Fomalhaut

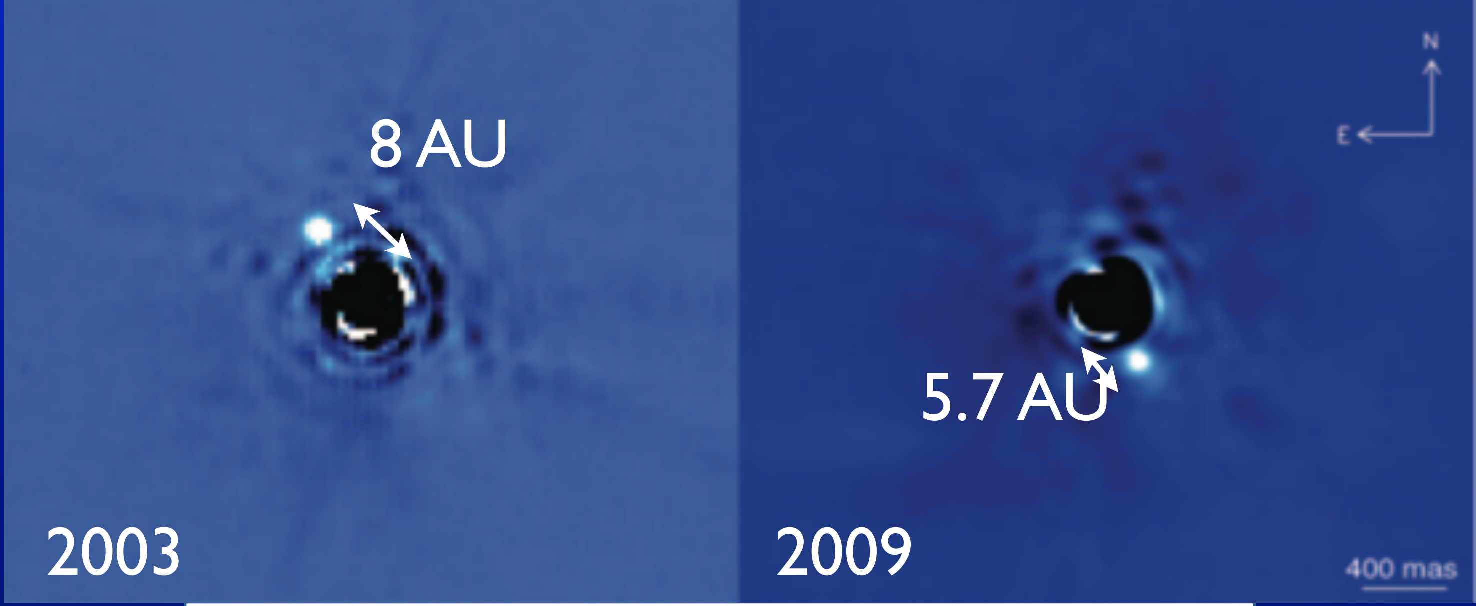
Eccentric planet begets eccentric ring

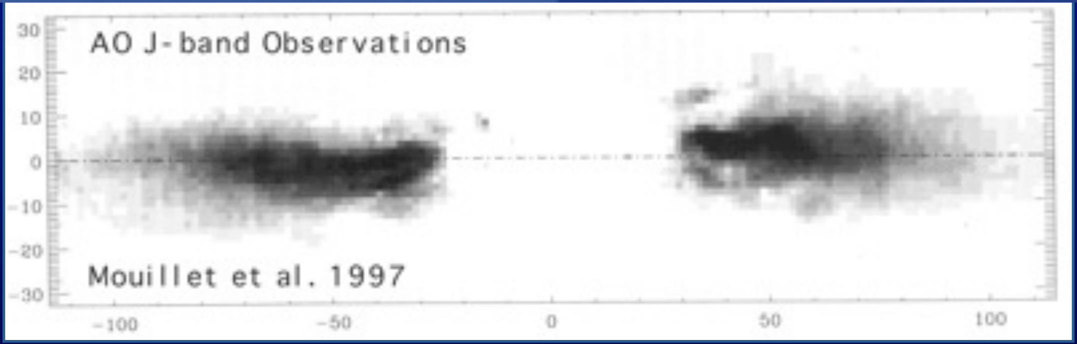


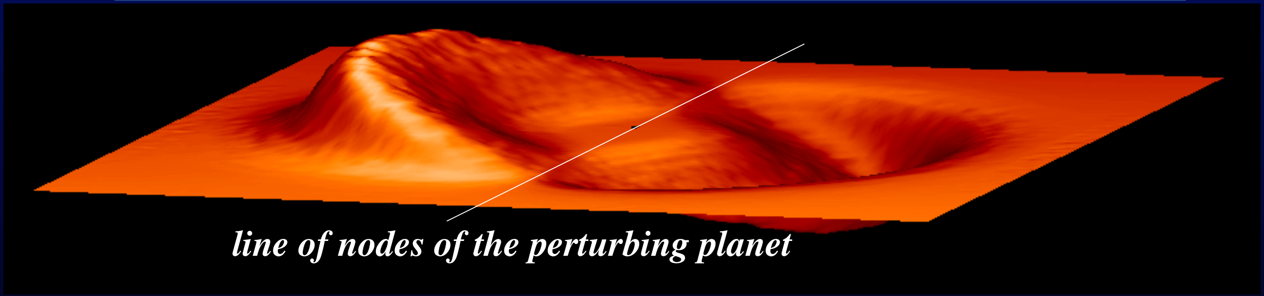


Equilibrium belt orbits are eccentric and aligned with the planet's orbit

beta Pic b





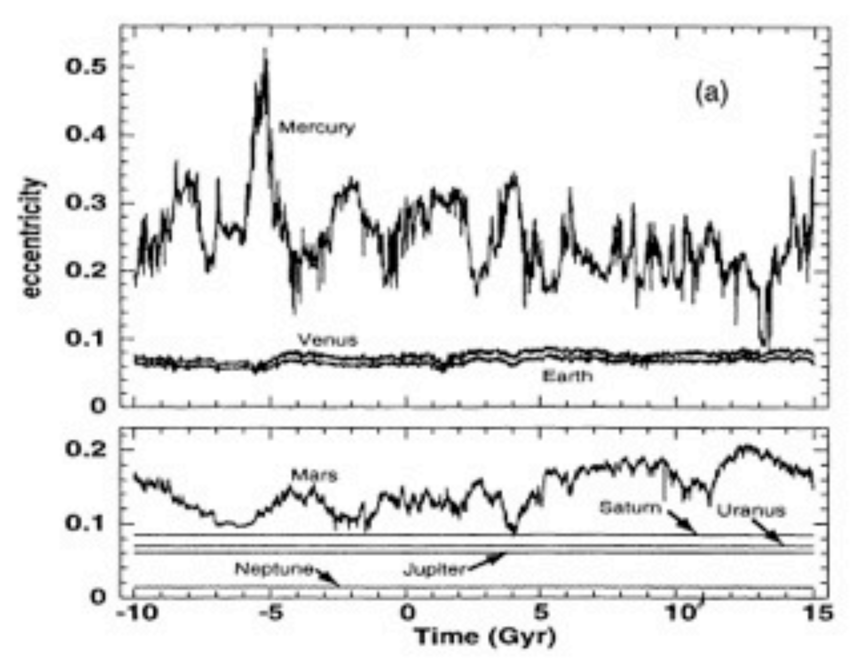


VLT 8-m with L' Adaptive Optics

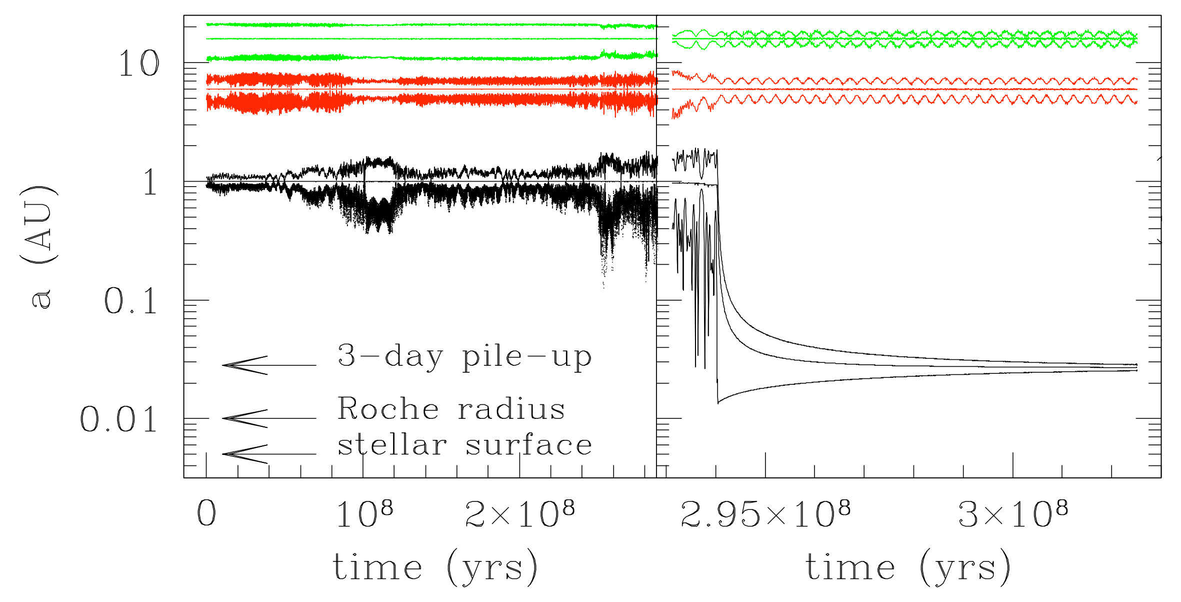
$L \sim 10^{-3.7} L_\odot$, $t \sim 12$ Myr → $M \sim 9\ M_J$

semimajor axis 8-15 AU; consistent with evolving vertical warp

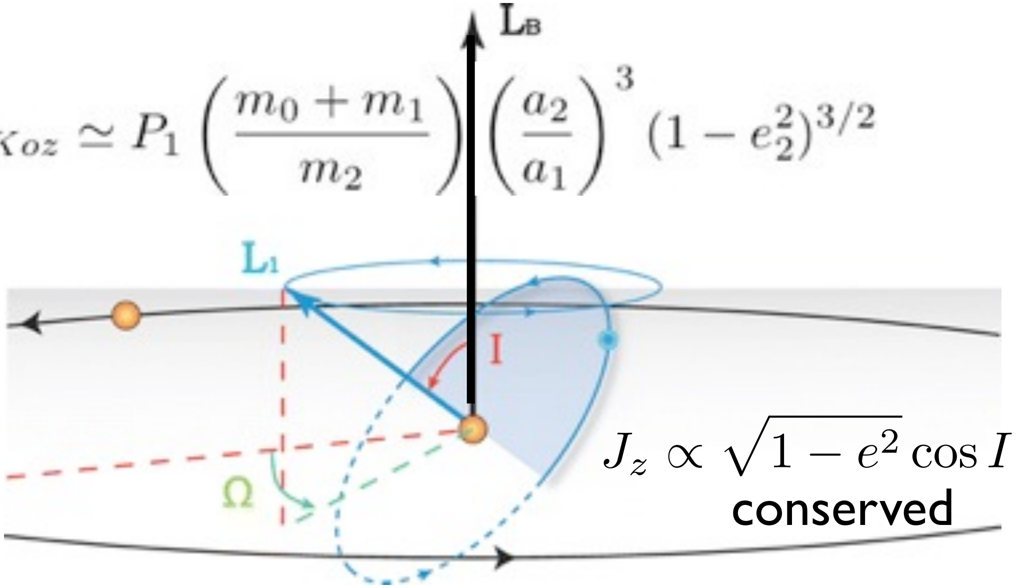
Mode amplitudes vary with time in nonlinear secular theory



Mercury's chaotic orbit
(Laskar 96)



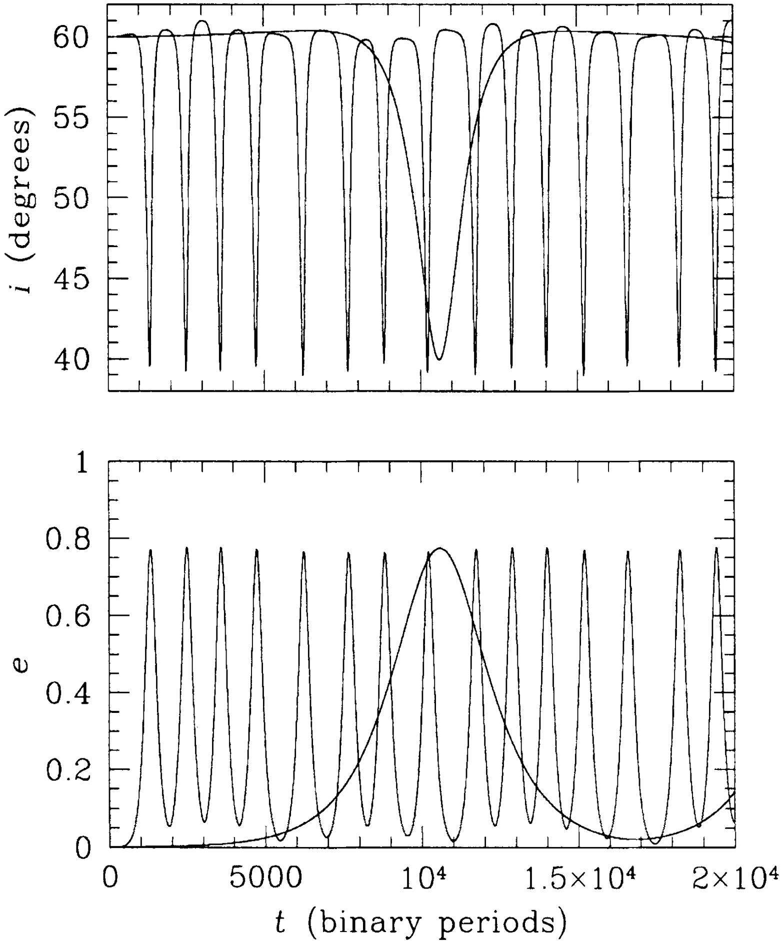
Production of
hot Jupiters in
multi-planet systems
by secular chaos
(Wu & Lithwick 10)

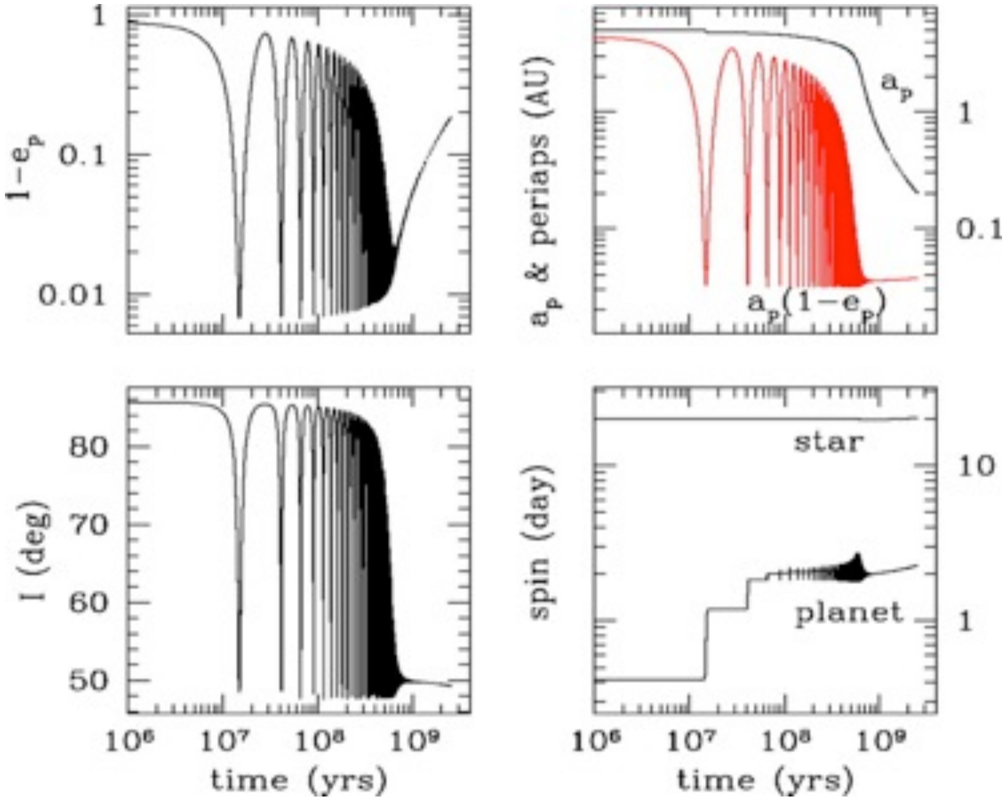


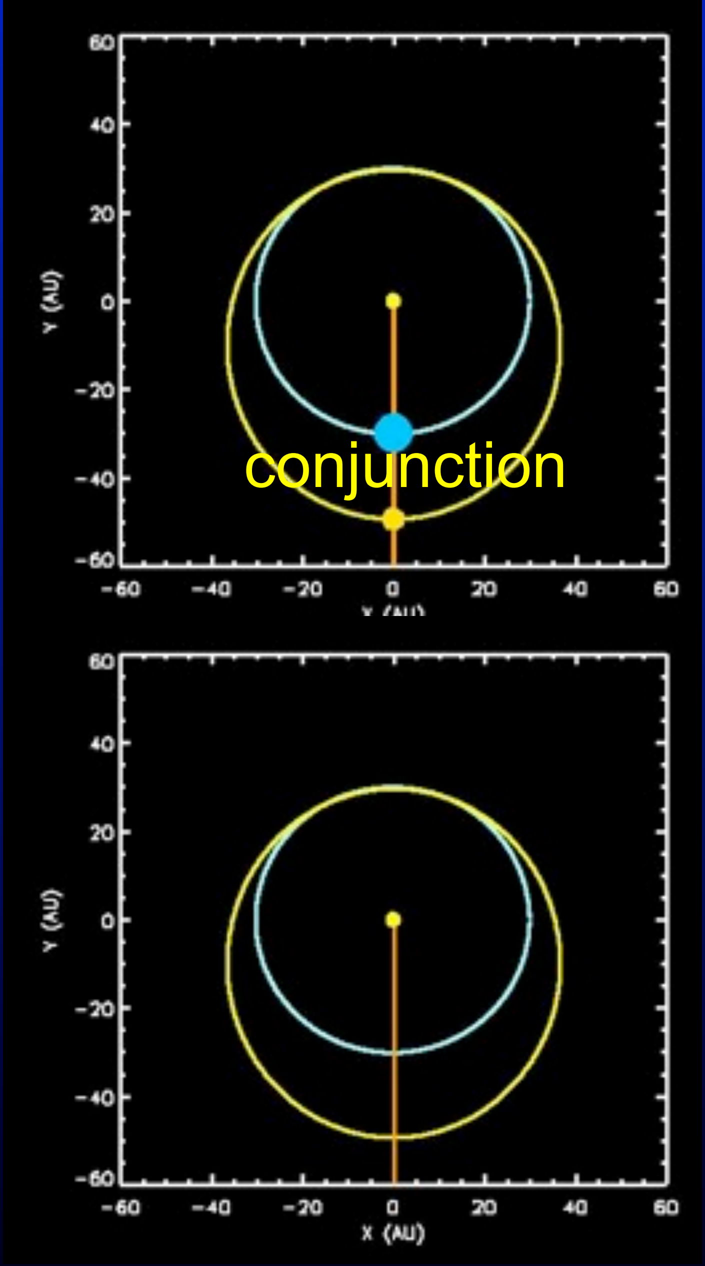
$$P_{Koz} \simeq P_1 \left(\frac{m_0 + m_1}{m_2}\right)\left(\frac{a_2}{a_1}\right)^3 (1 - e_2^2)^{3/2}$$

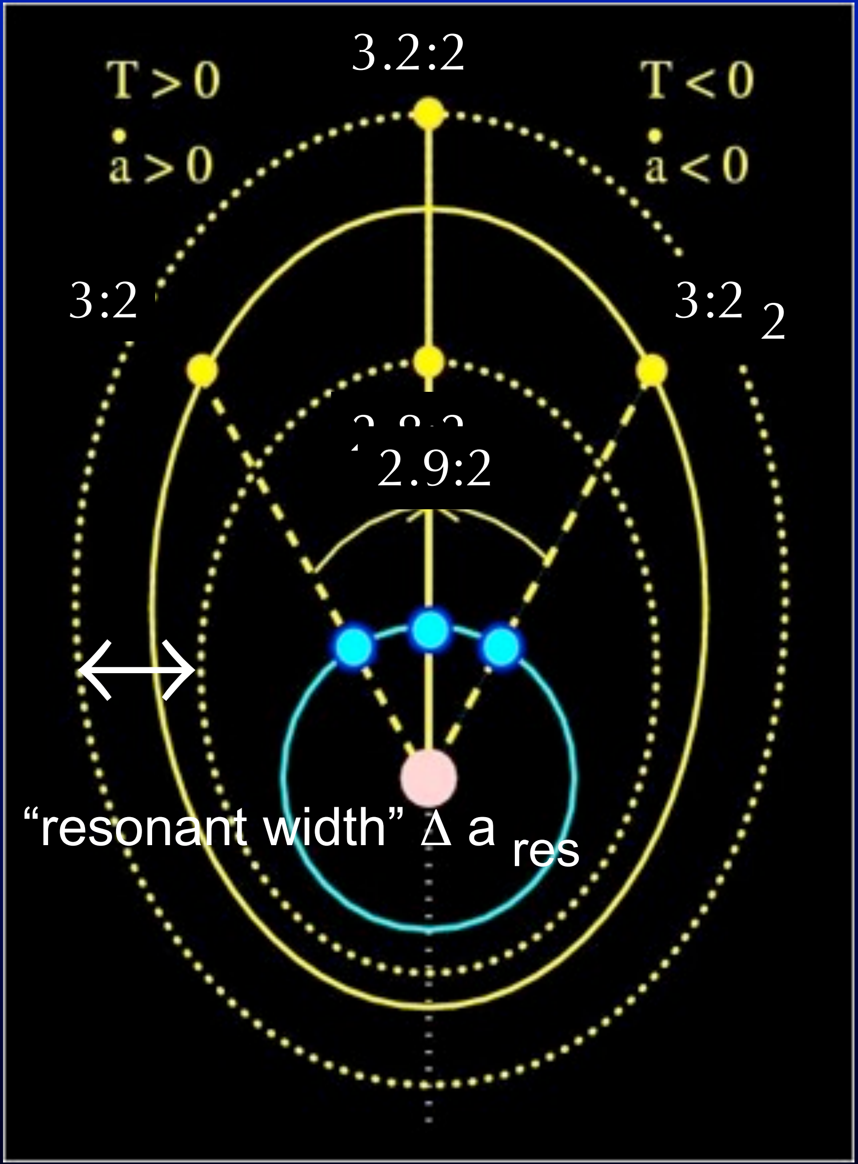
$$J_z \propto \sqrt{1 - e^2} \cos I$$

conserved

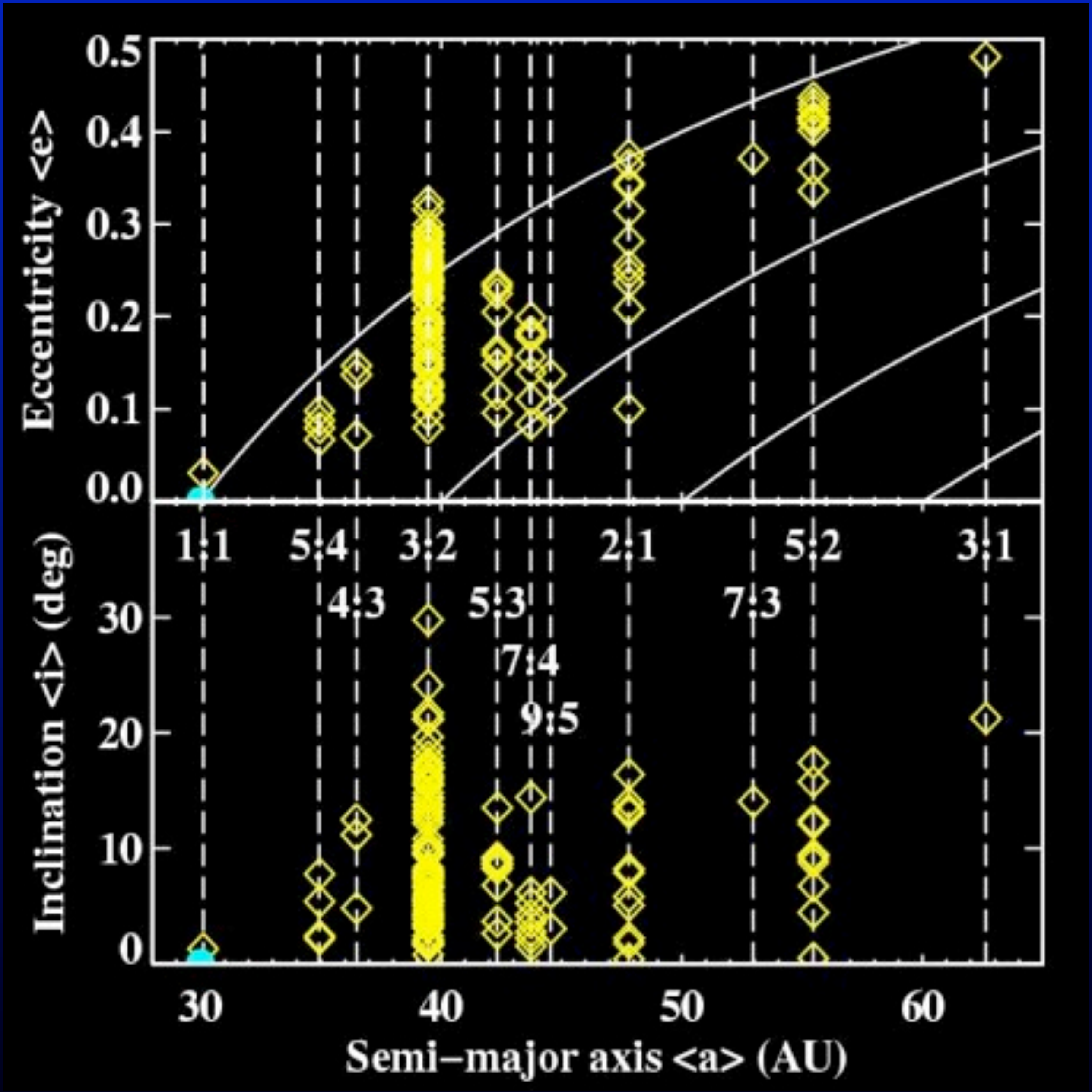






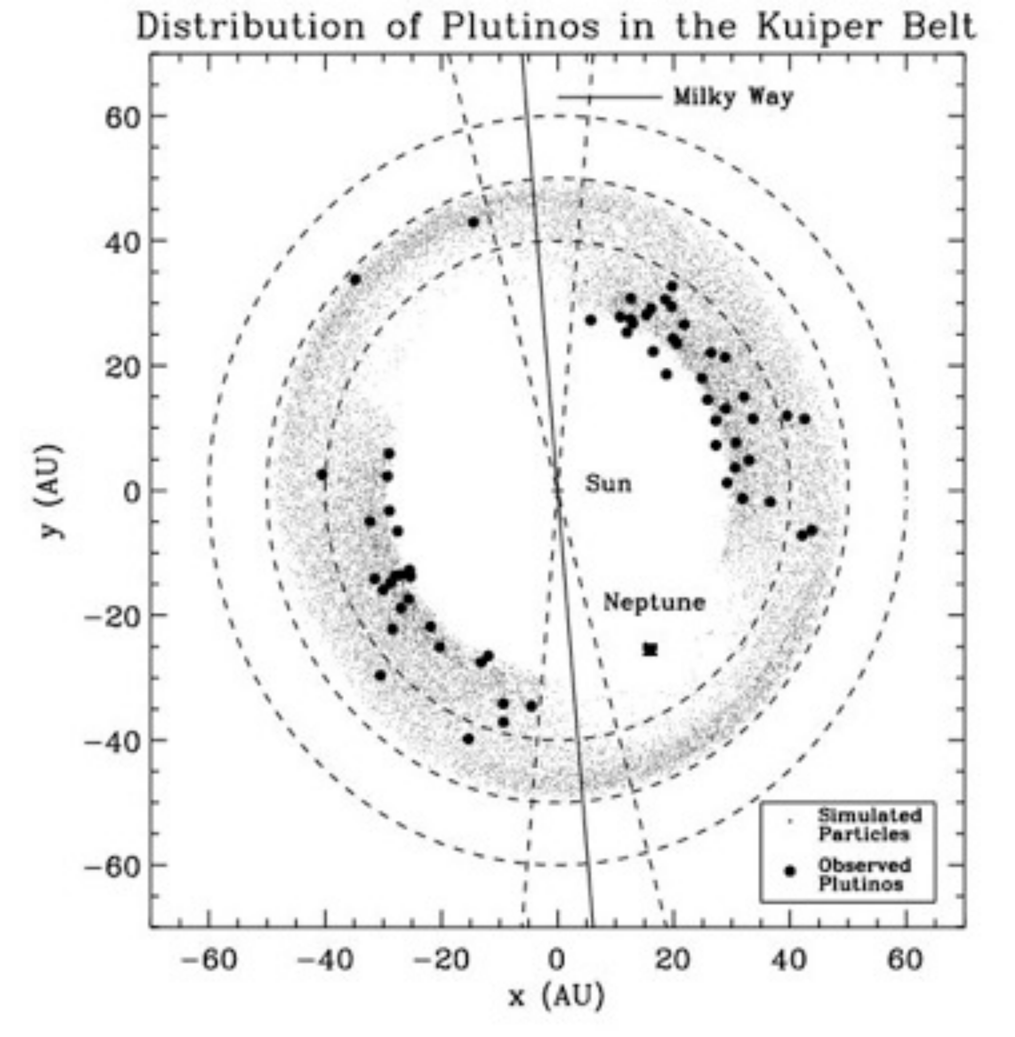


Neptune-Pluto Orbit-Orbit Resonance



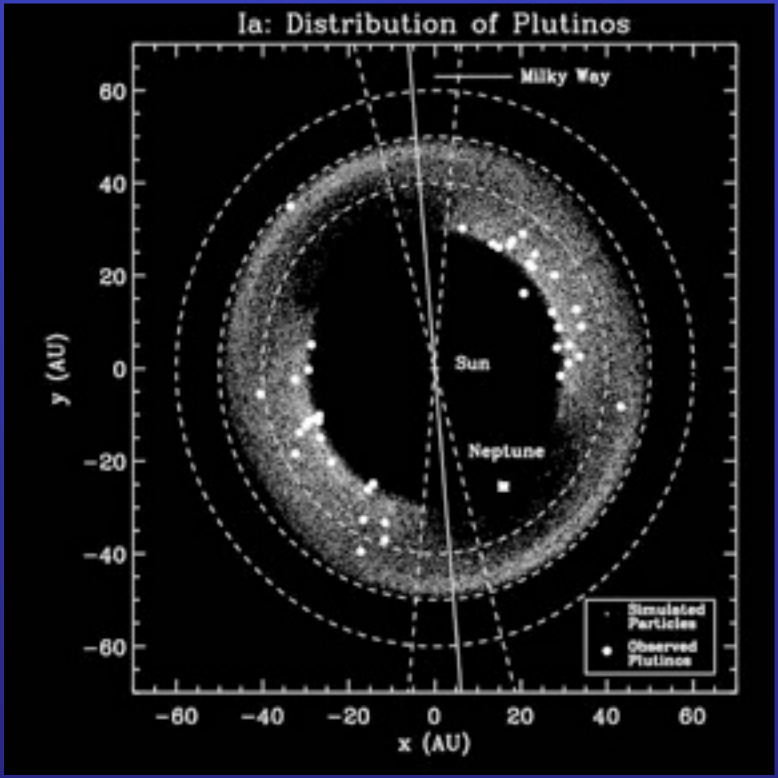
Resonant KBOs (~26%)

Plutino (3:2) Snapshot

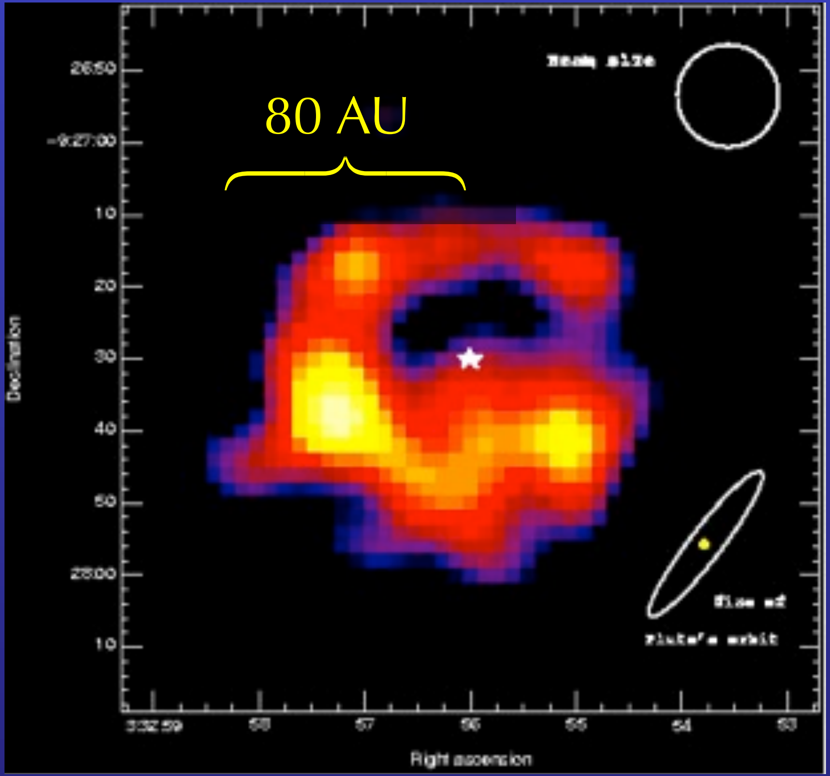


Wave pattern rotates rigidly with Neptune

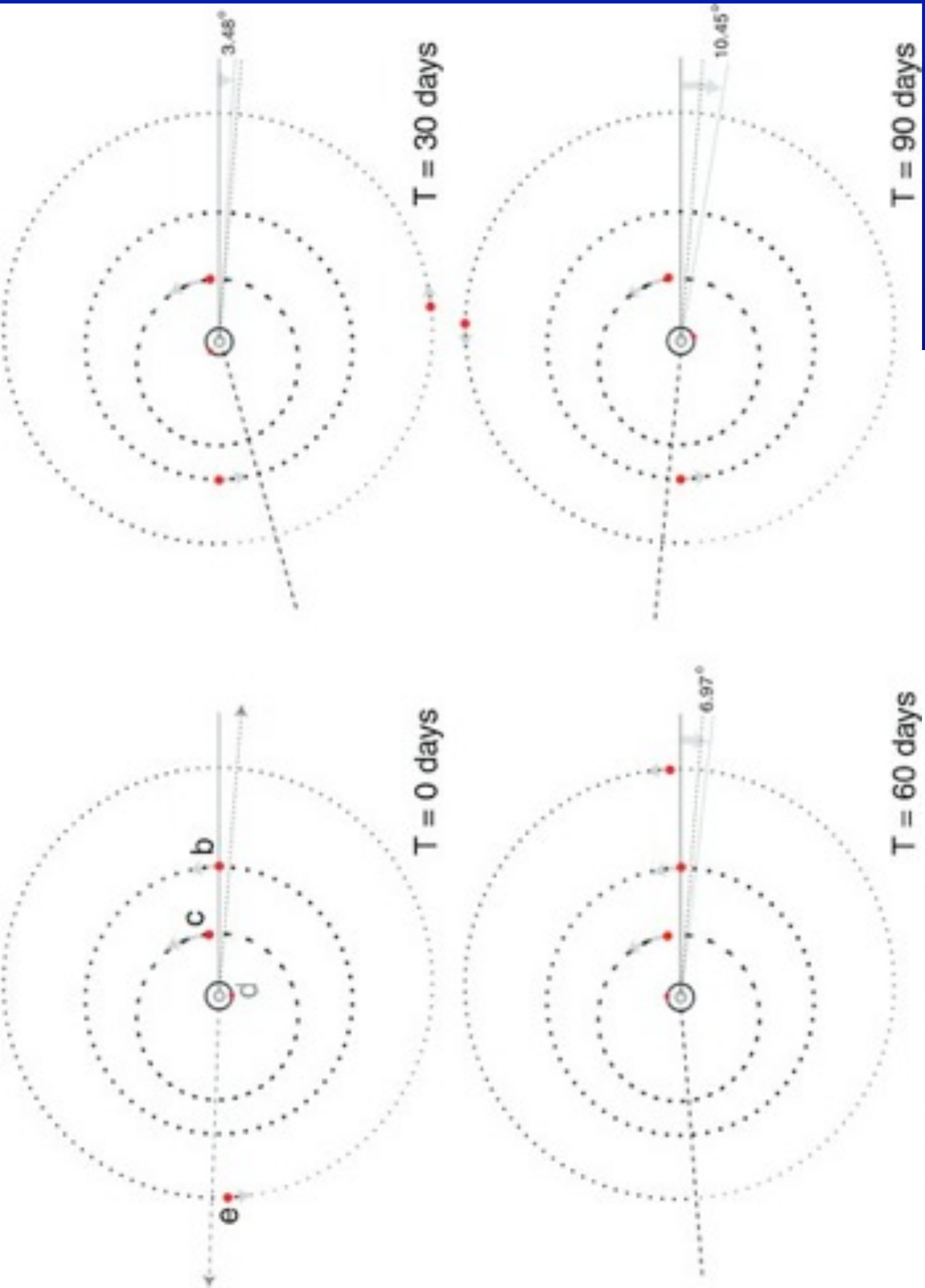
Resonant clustering of Plutinos (3:2) in the Kuiper Belt

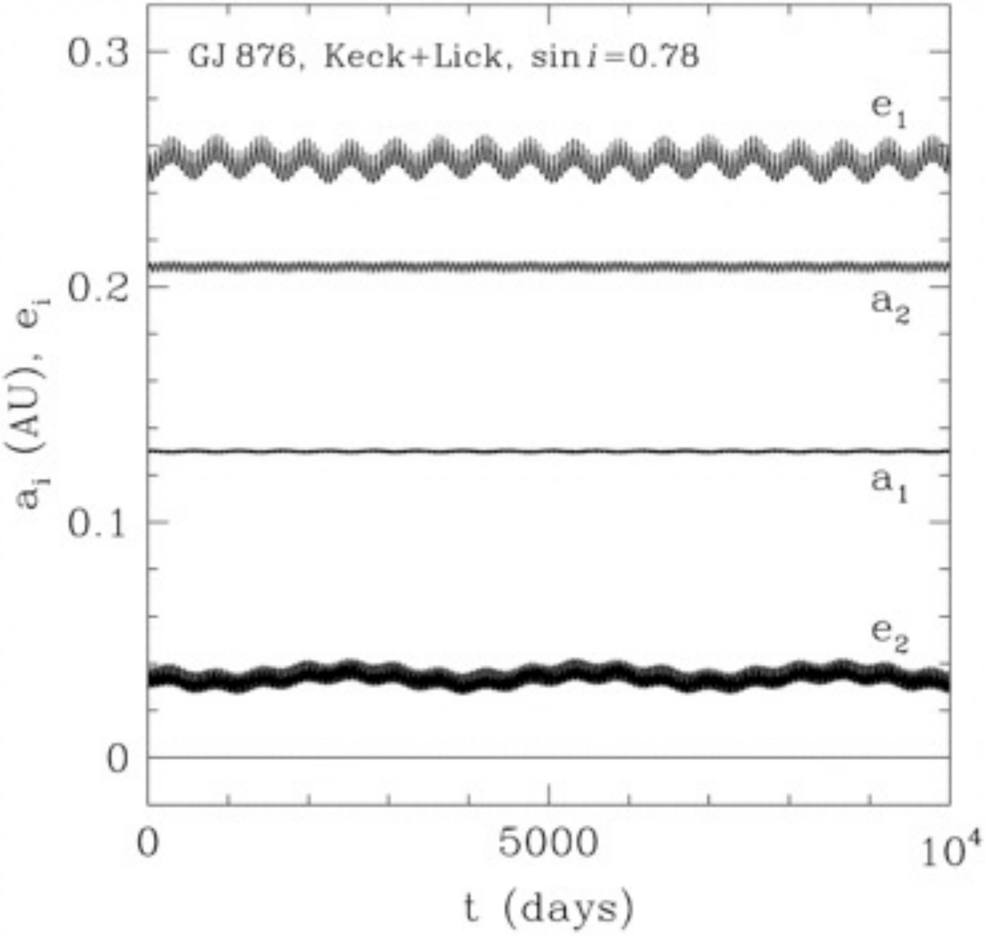


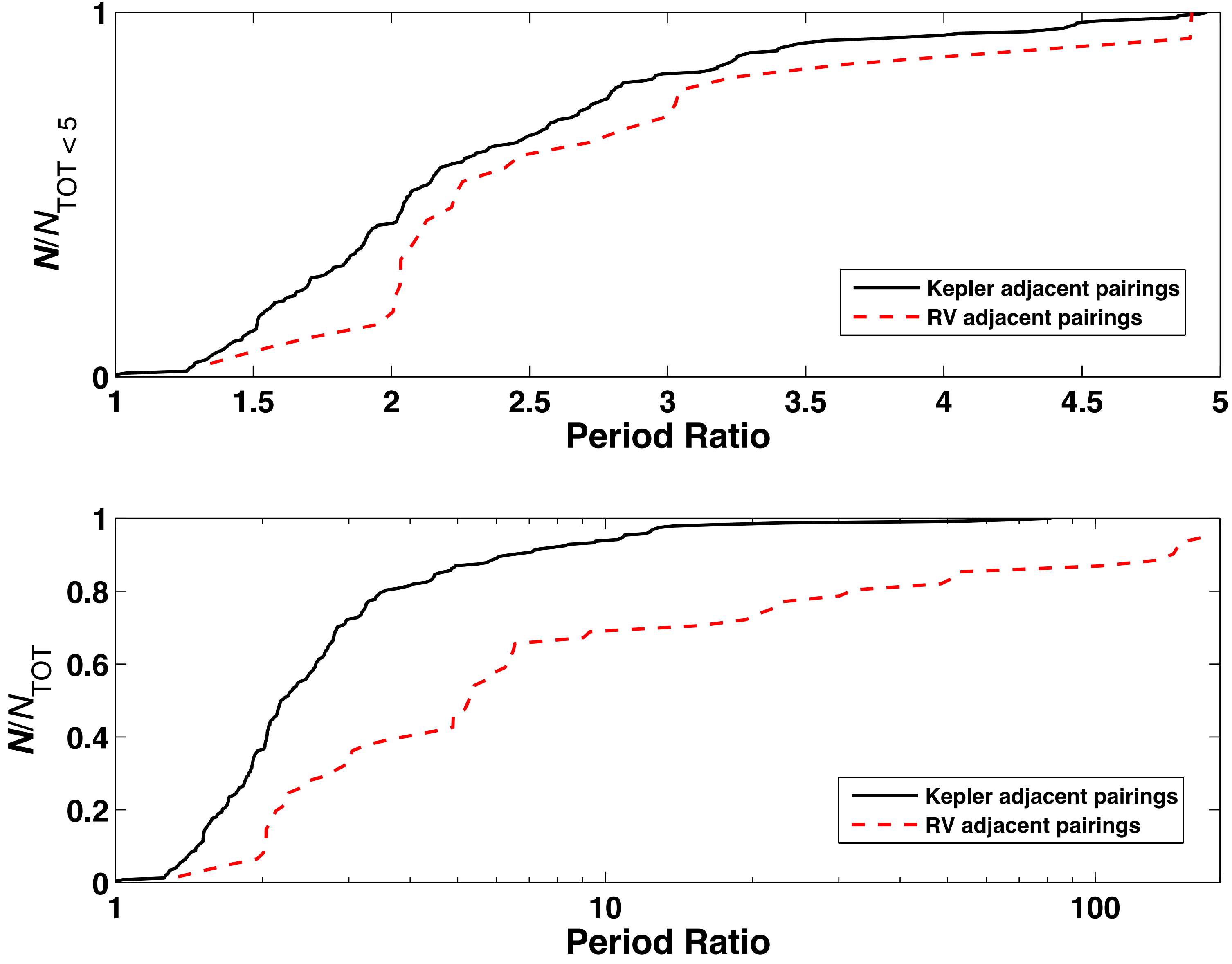
Epsilon Eridani

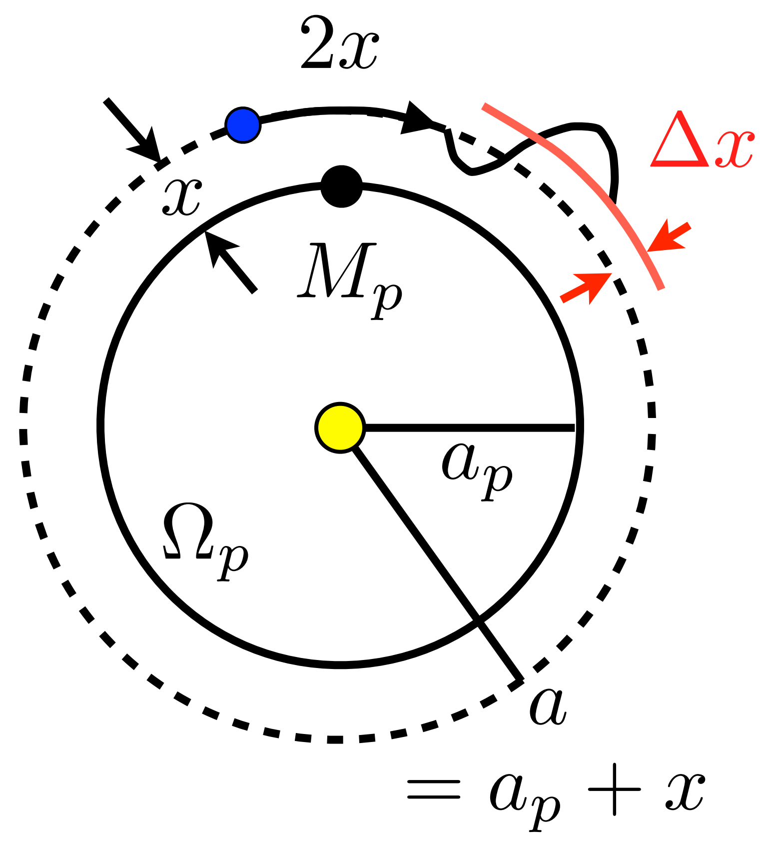


GJ 876:
Resonant
Planetary System






$N/N_{TOT < 5}$
1
0
1
1.5
2
2.5
3
3.5
4
4.5
5
Period Ratio
Kepler adjacent pairings
RV adjacent pairings
N/N_{TOT}
1
0.8
0.6
0.4
0.2
0
1
10
100
Period Ratio
Kepler adjacent pairings
RV adjacent pairings



Deriving the chaotic zone width

I. The kick at conjunction

$$x \ll a$$

Kick in eccentricity Δe

$$\Delta e \sim \frac{\Delta v}{v} \sim \frac{1}{v}\frac{GM_p}{x^2}\Delta t$$

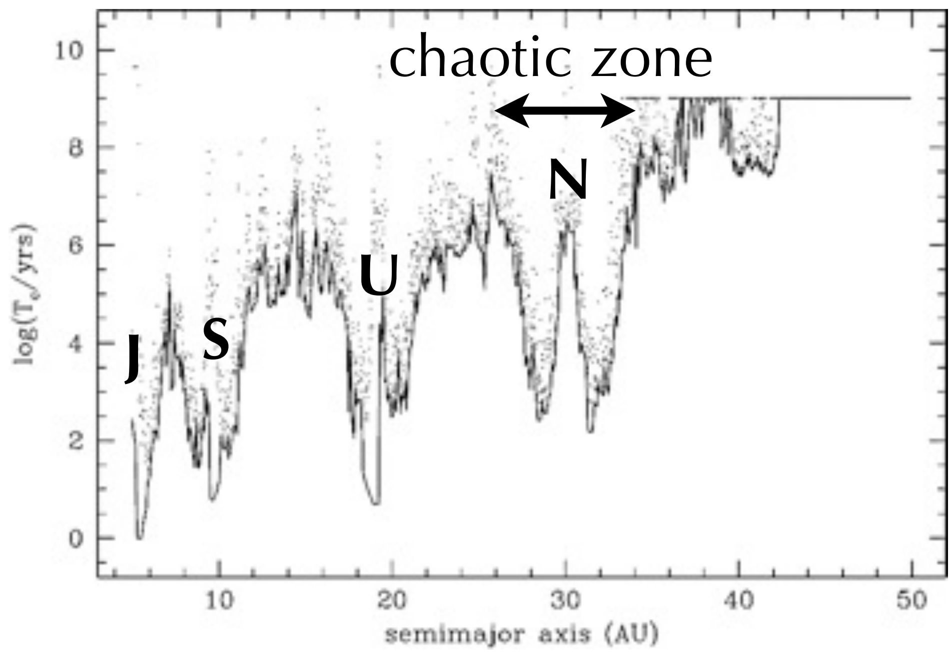
$$\sim \frac{M_p}{M_*}\left(\frac{a}{x}\right)^2$$

Kick in semimajor axis Δx

Use Jacobi constant: $C_J \approx -\frac{GM_*}{2(a+x)} - \Omega_p\sqrt{M_*(a+x)(1-e^2)}$

$$\Rightarrow \frac{x\Delta x}{a^2} \sim (\Delta e)^2 \Rightarrow \frac{\Delta x}{a} \sim \left(\frac{M_p}{M_*}\right)^2\left(\frac{a}{x}\right)^5$$

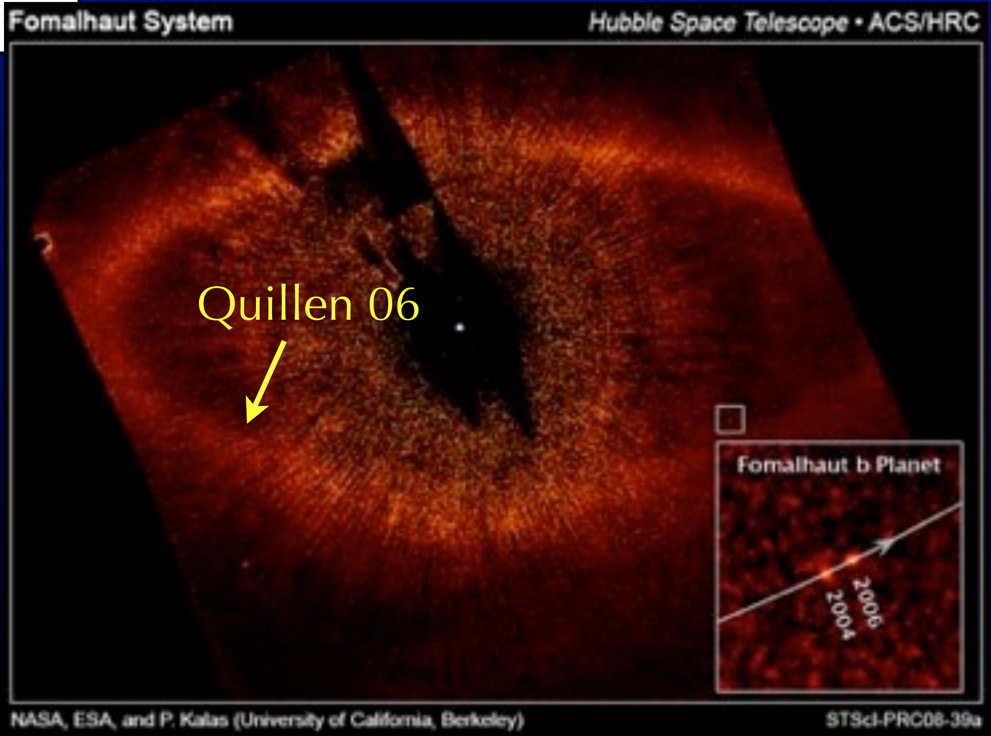
Constraining a_p and M_p using the sharp inner belt edge

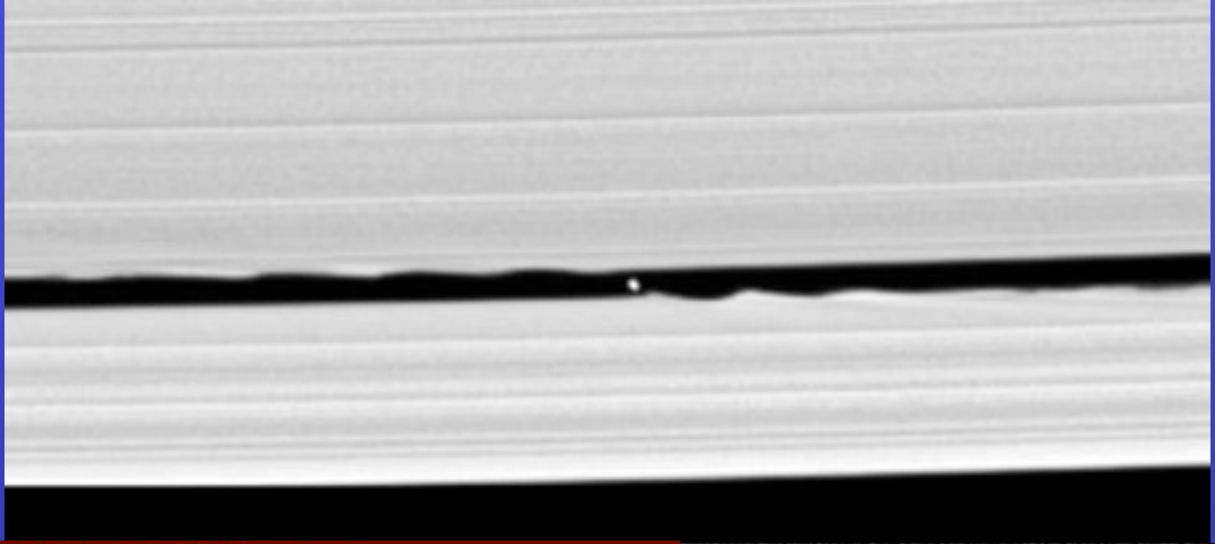


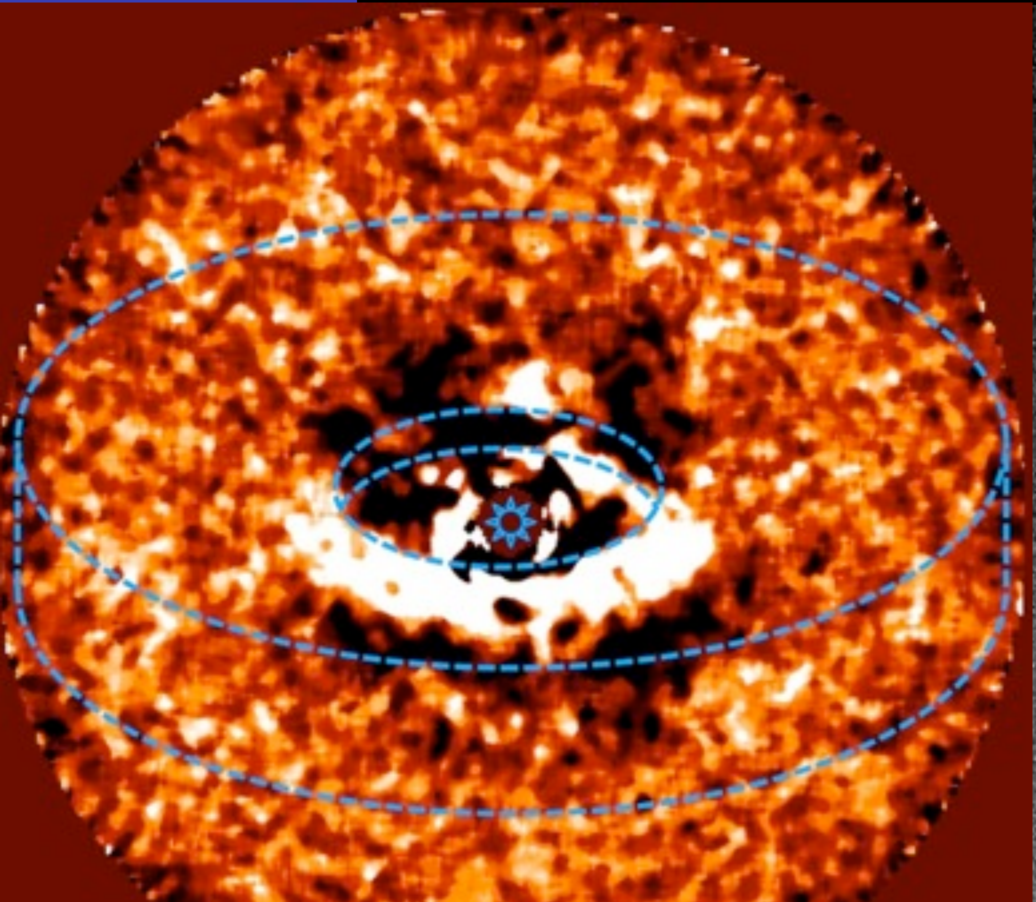
Lecar et al. 2001

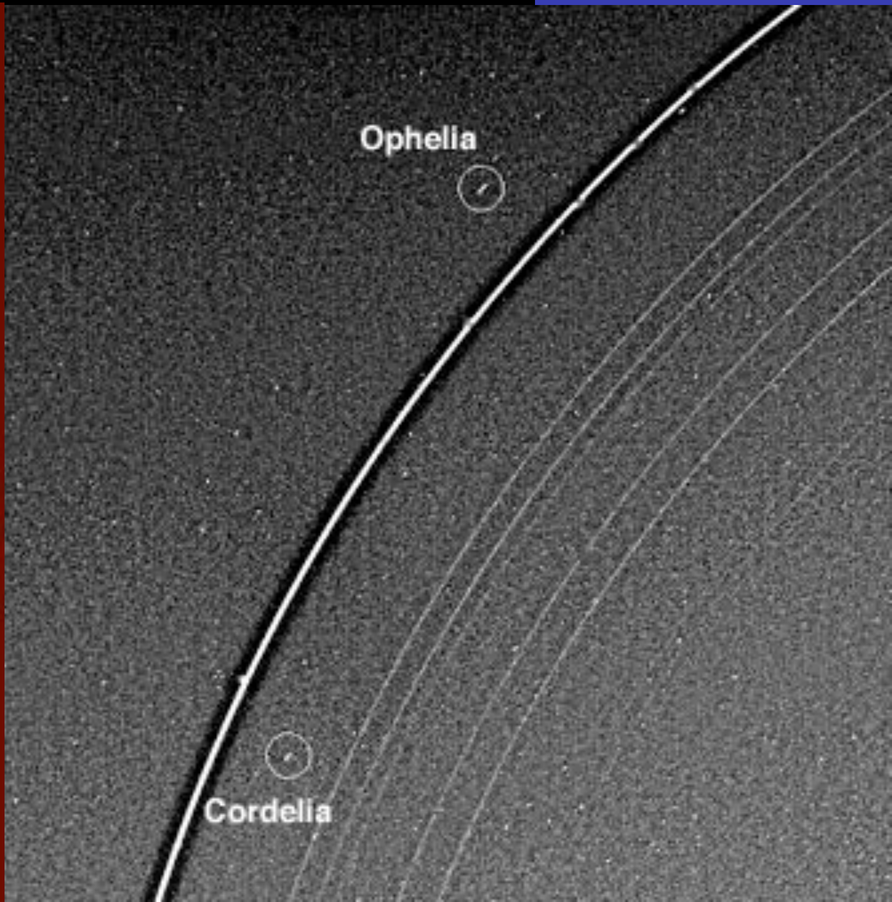
Inner belt edge = Outer edge of planet's "chaotic zone"

Chaotic zone width ~ $(M_{planet}/M_{star})^{2/7}\ a_{planet}$

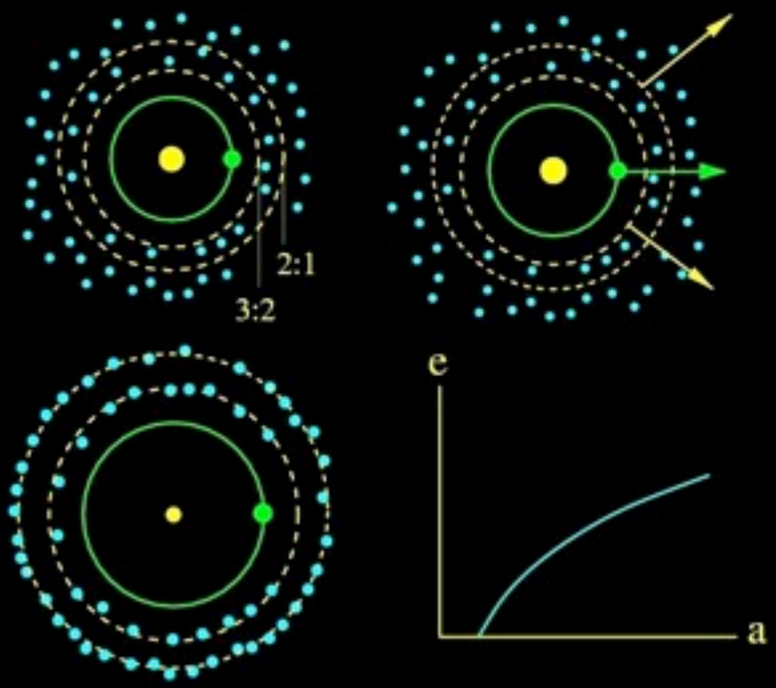


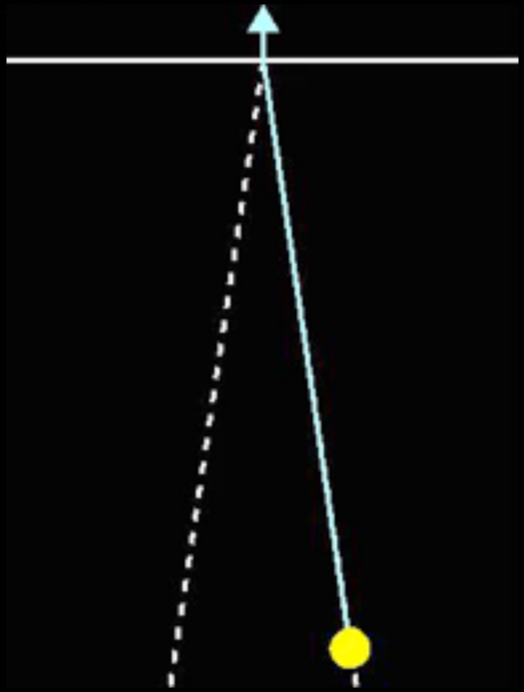


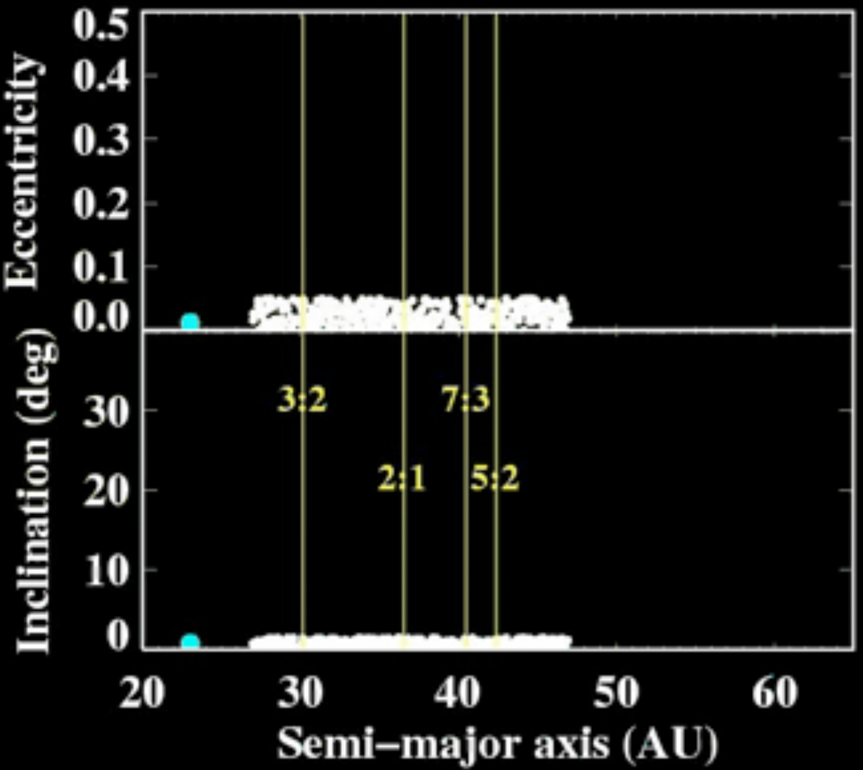



Ophelia
Cordelia

Resonance Sweeping



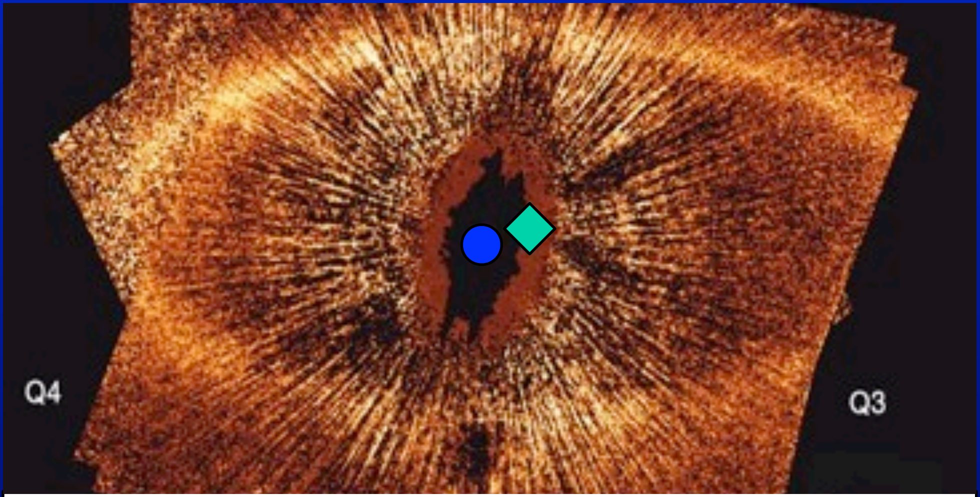


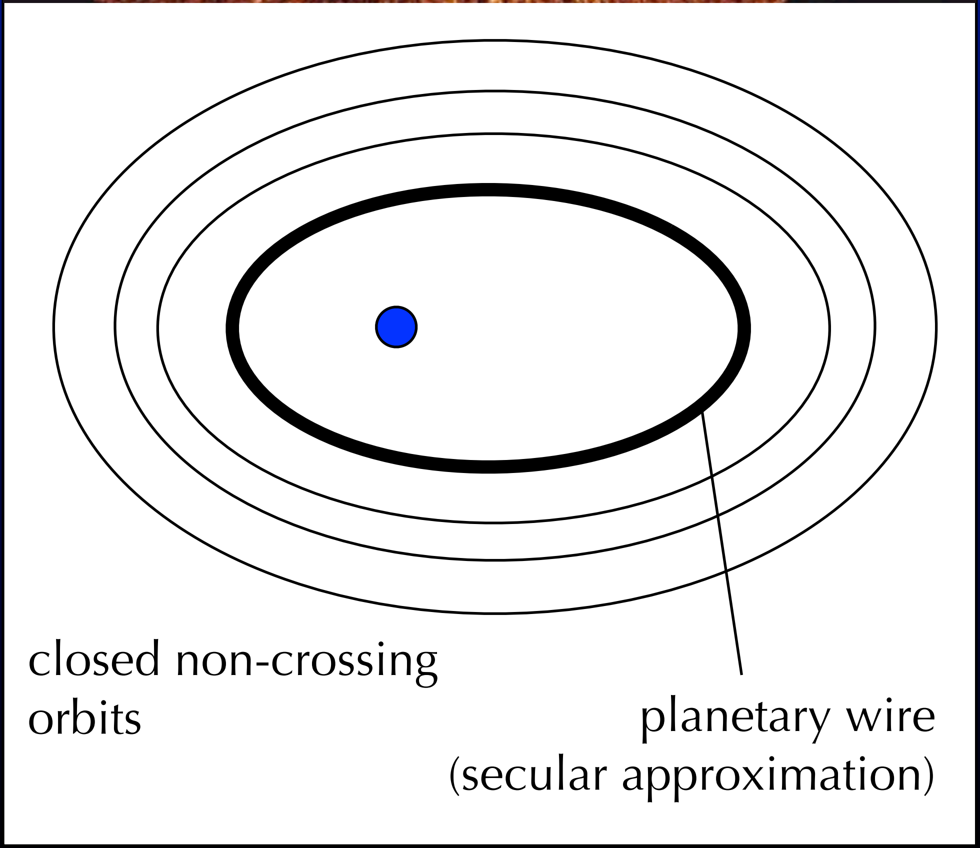


$\Gamma = \oint p\, dq =$ conserved

$\Gamma_1 =$ adiabatic invariant over synodic period

$\Gamma_2 =$ adiabatic invariant over libration period





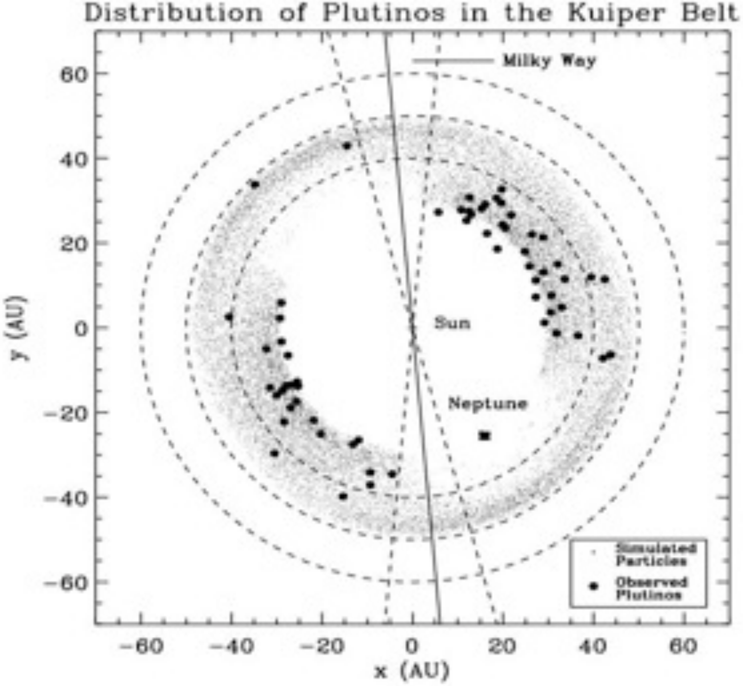
Dissipative relaxation of parent bodies onto non-crossing (forced eccentric) orbits

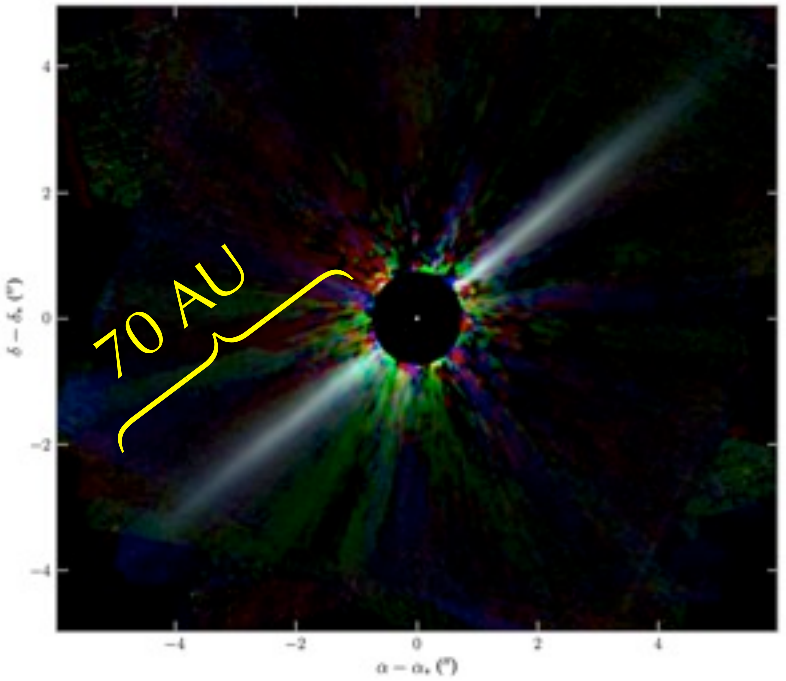
Relaxation occurs during:

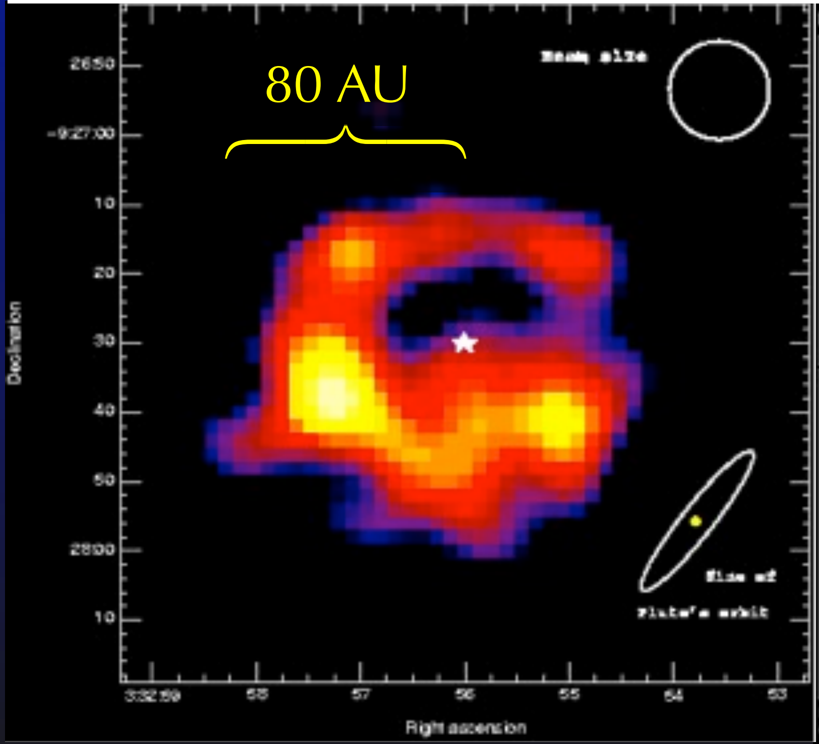
- Present-day collisional cascade
- Prior coagulation

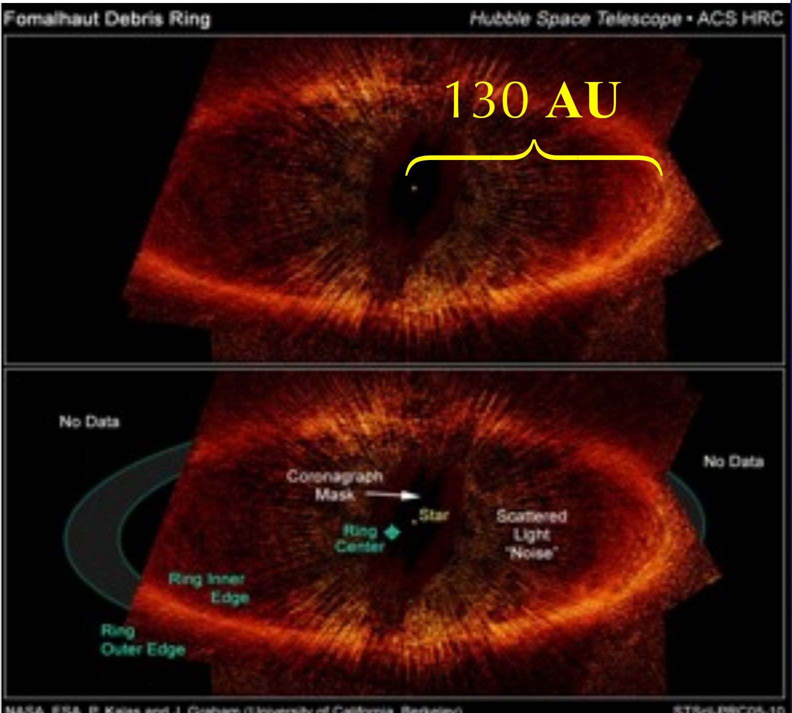
$$e_{\rm forced}(a) = \frac{b_{3/2}^{(2)}(a_{\rm planet}/a)}{b_{3/2}^{(1)}(a_{\rm planet}/a)} e_{\rm planet}$$

Wyatt et al. 99
Kalas et al. 05



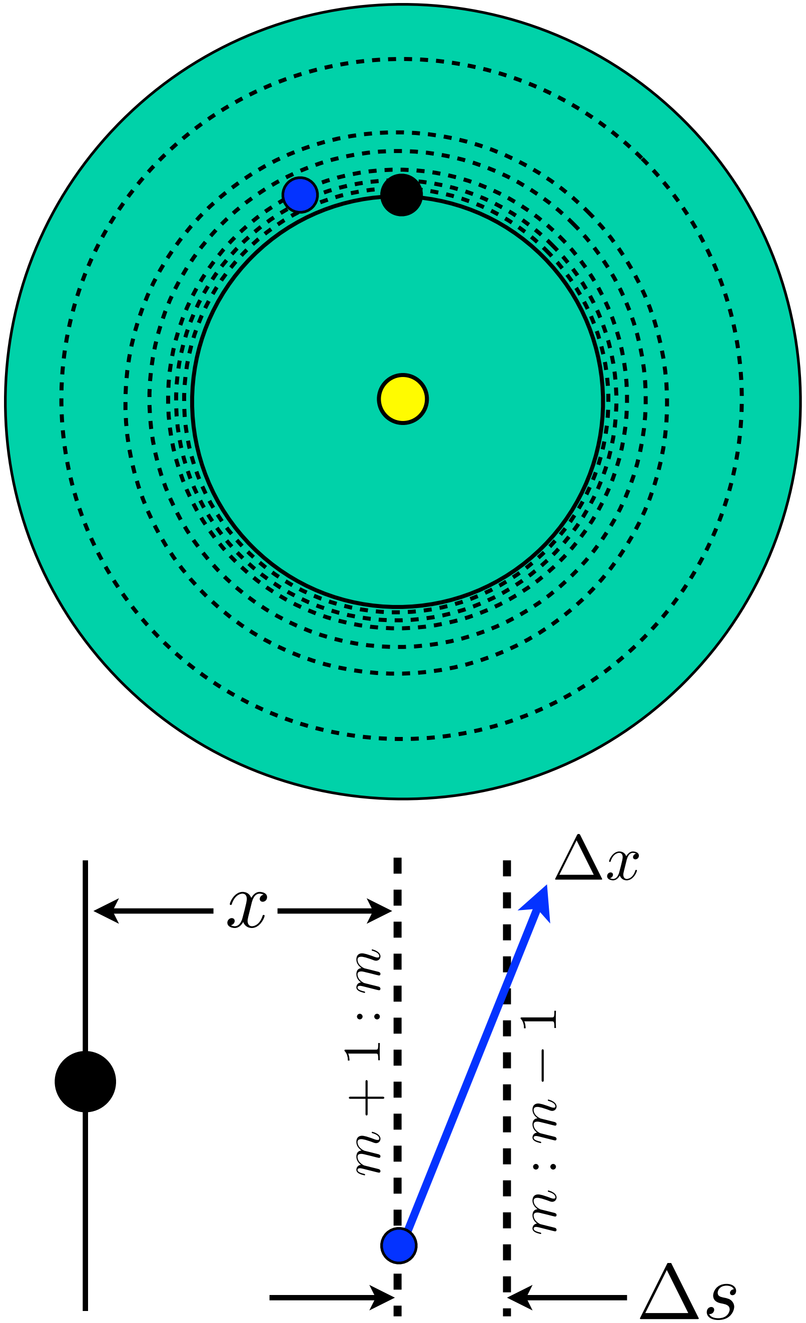






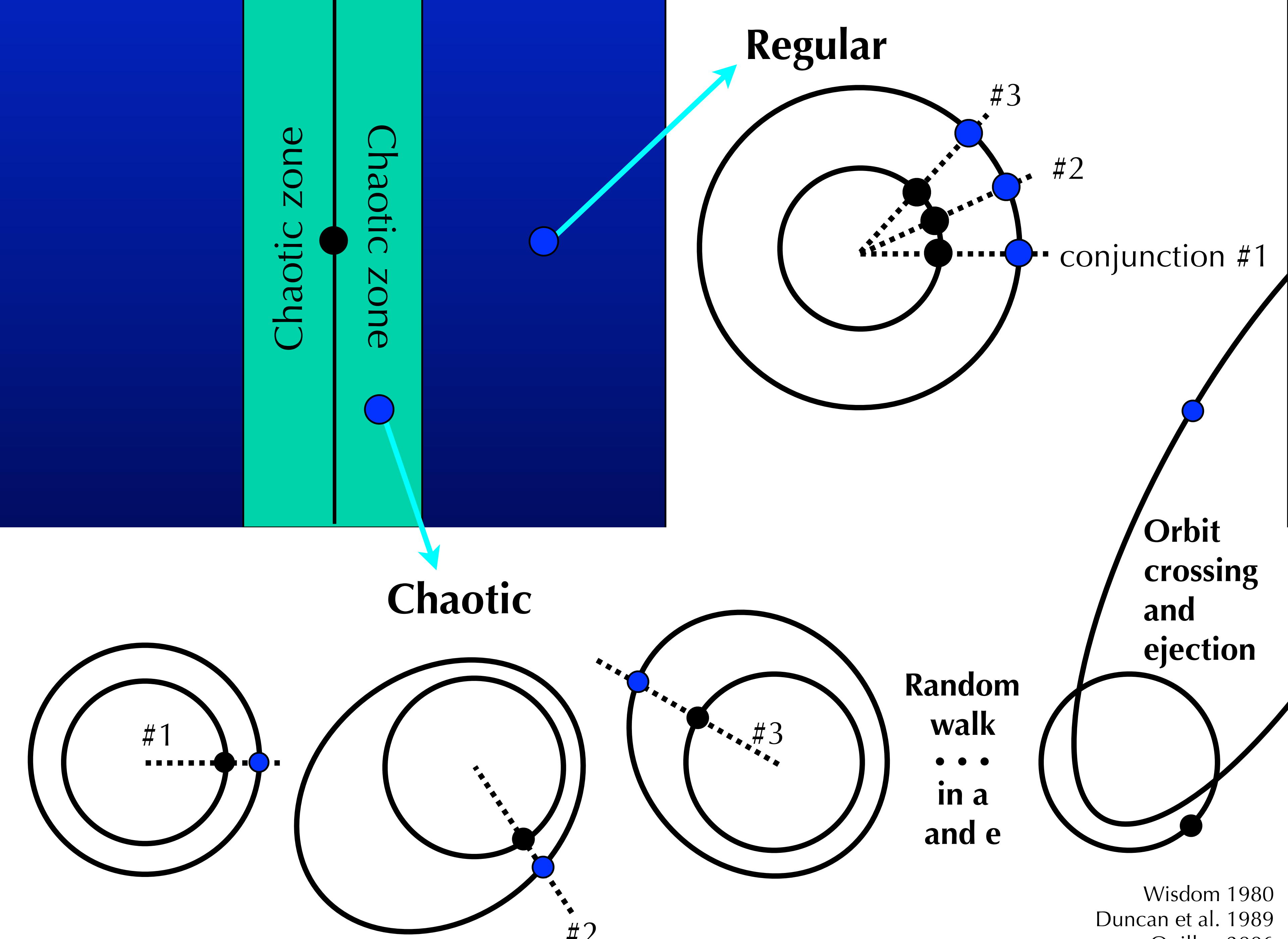
NASA, ESA, P. Kalas and J. Graham (University of California, Berkeley) and M. Clampin (NASA/GSFC)

STScI-PRC05-10



Planetary chaotic zone

= Region where first-order resonances overlap



Regular

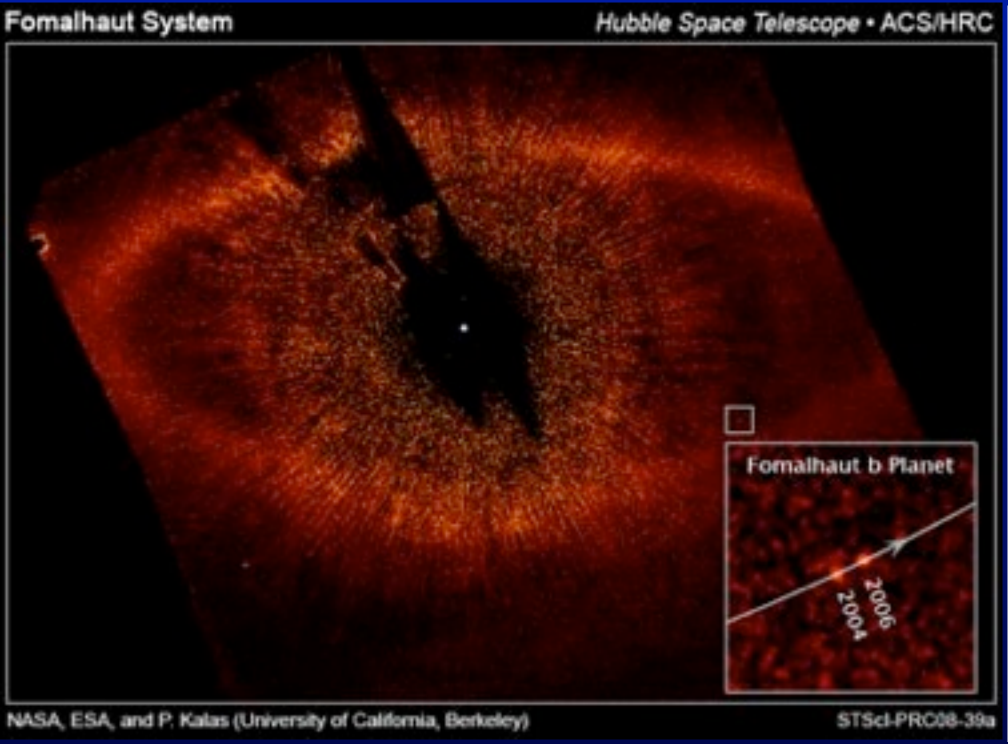
Chaotic

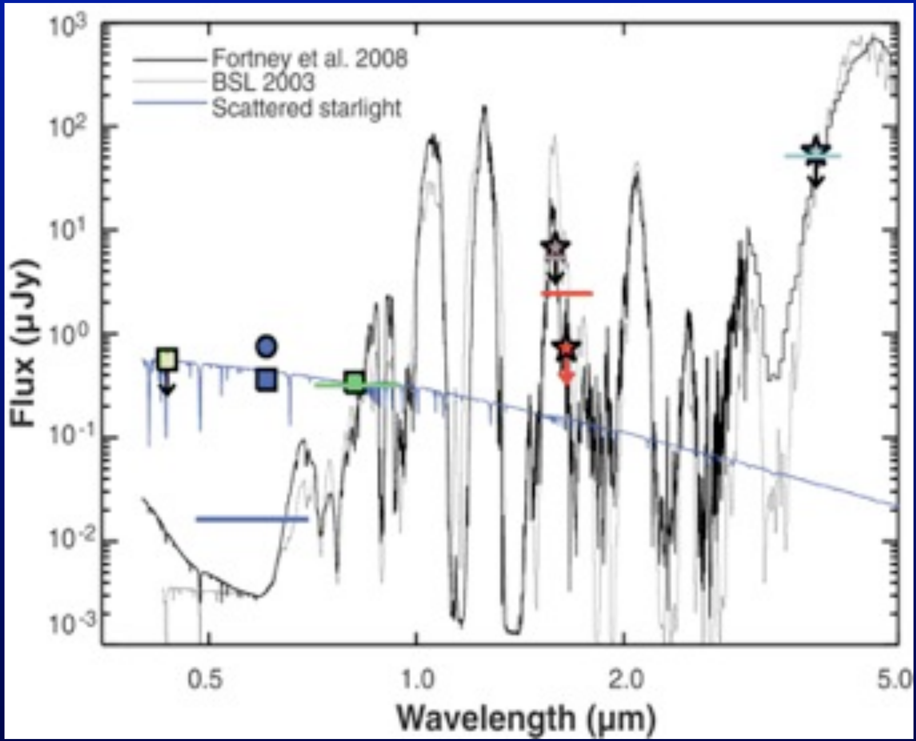
Random walk
. . .
in a and e

Orbit crossing and ejection

Wisdom 1980
Duncan et al. 1989
Quillen 2006

Candidate planet (0.5 Jupiter mass)





not confirmed: only 2 epochs

not thermal emission from planetary atmosphere

40 R_J reflective dust disk?

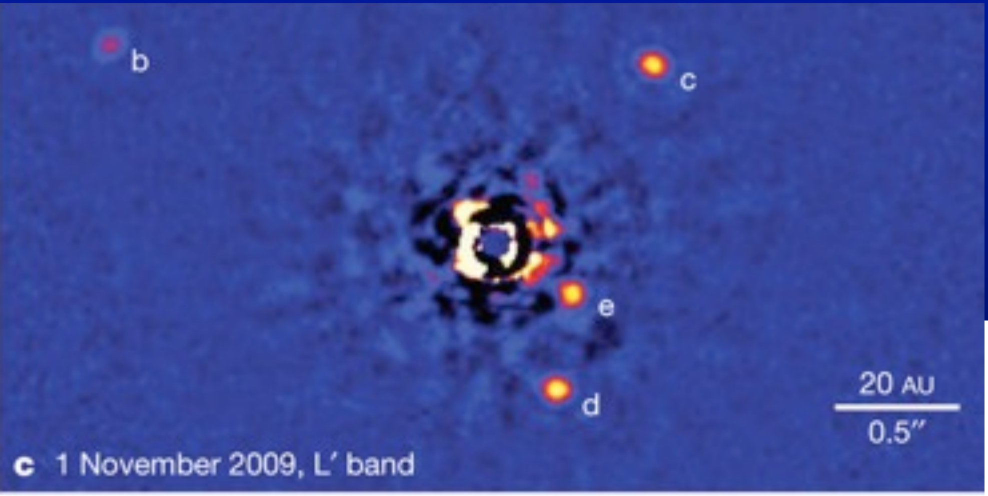
Variable Hα emission?

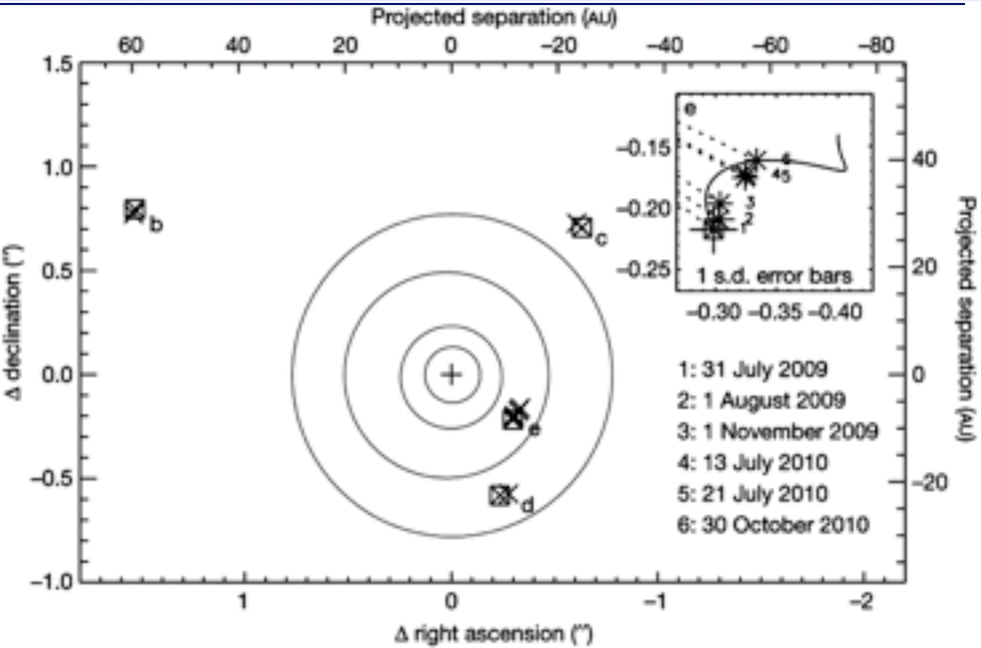
HR 8799
A-type star 30-60 Myr old
with 4 Super-Jupiters

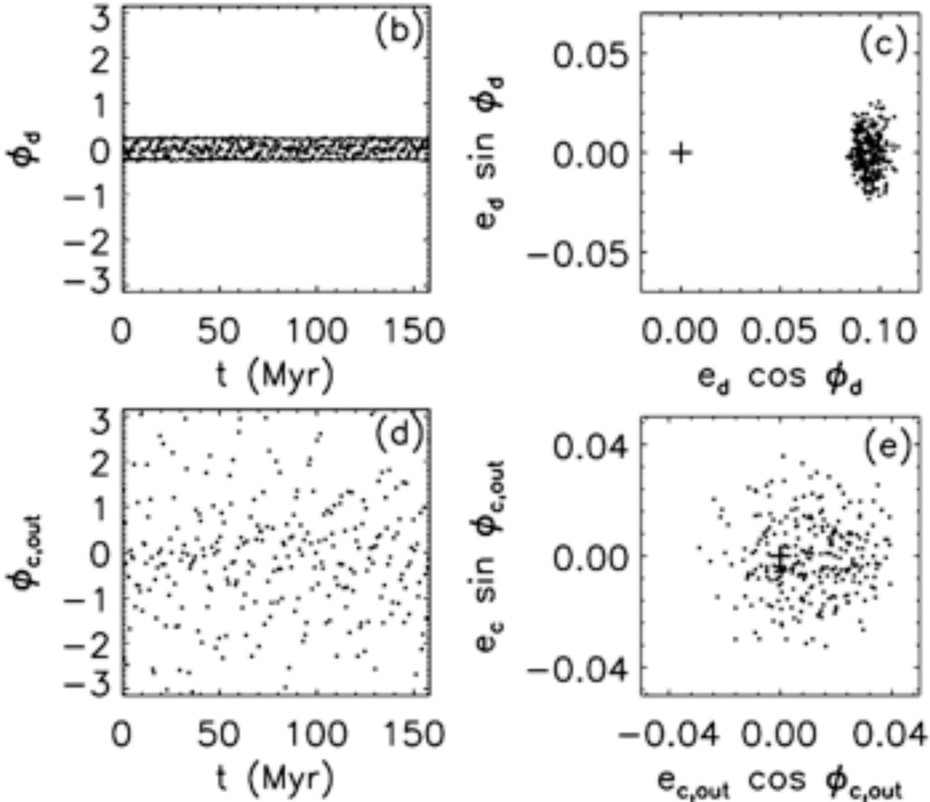
Orbital resonances afford stability
d:c = 2:1 resonance

Other possibilities include
d:c:b = 4:2:1
e:d:c = 4:2:1

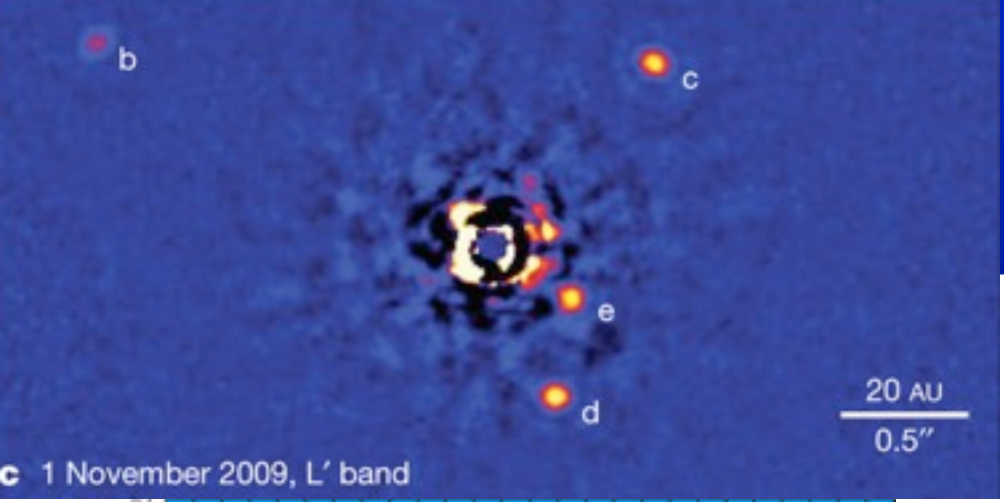
dynamical masses < 20 Jupiter masses each

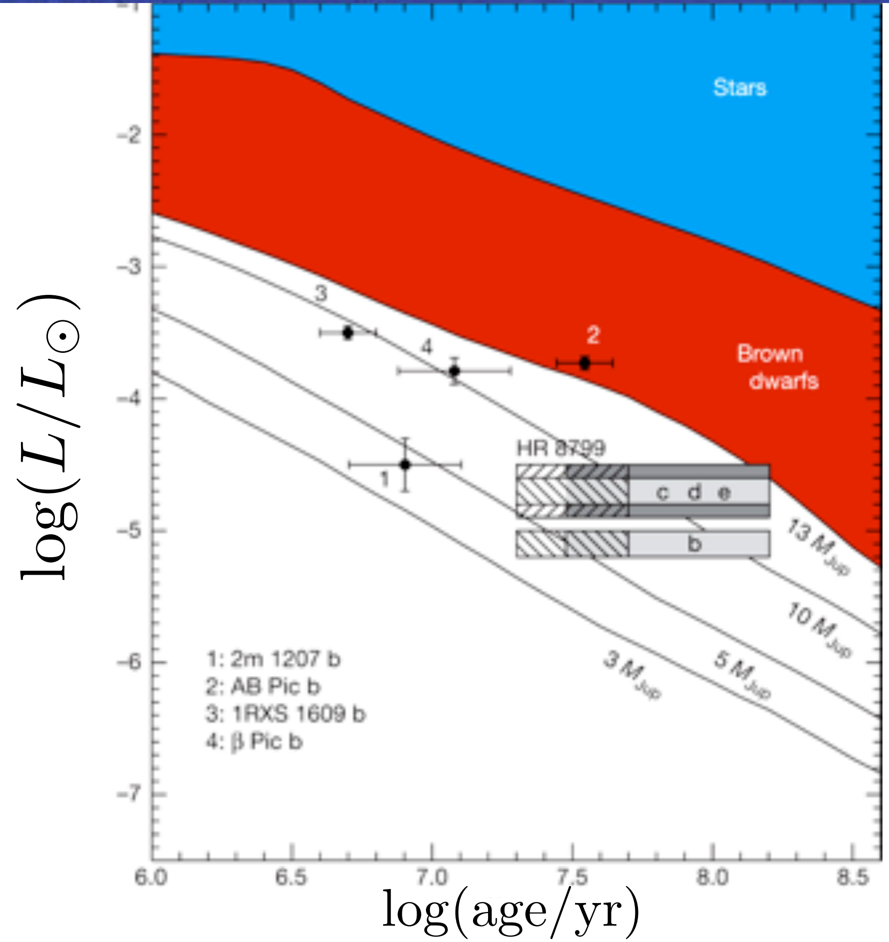


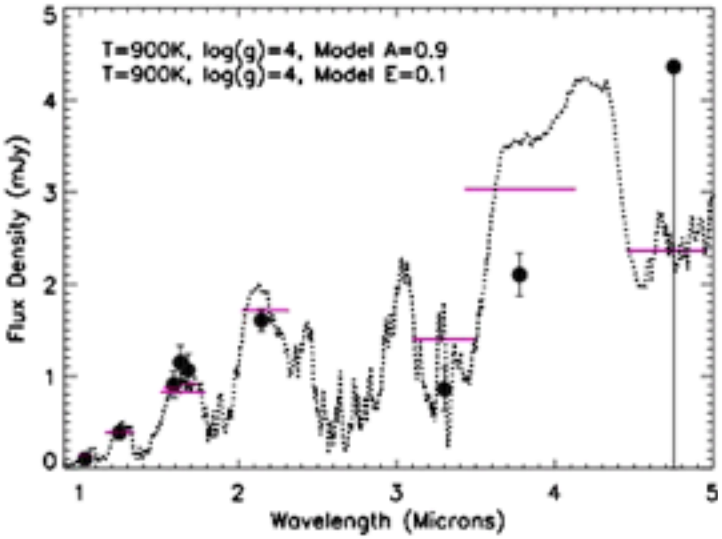




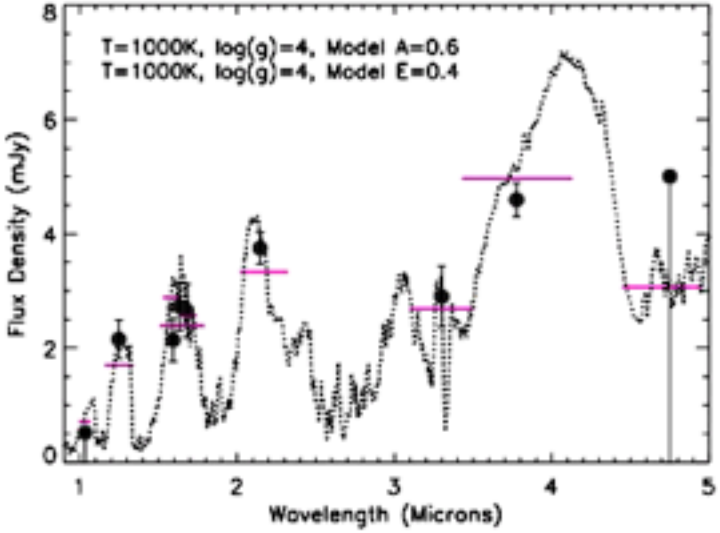
Cloudy spectra unlike brown dwarfs





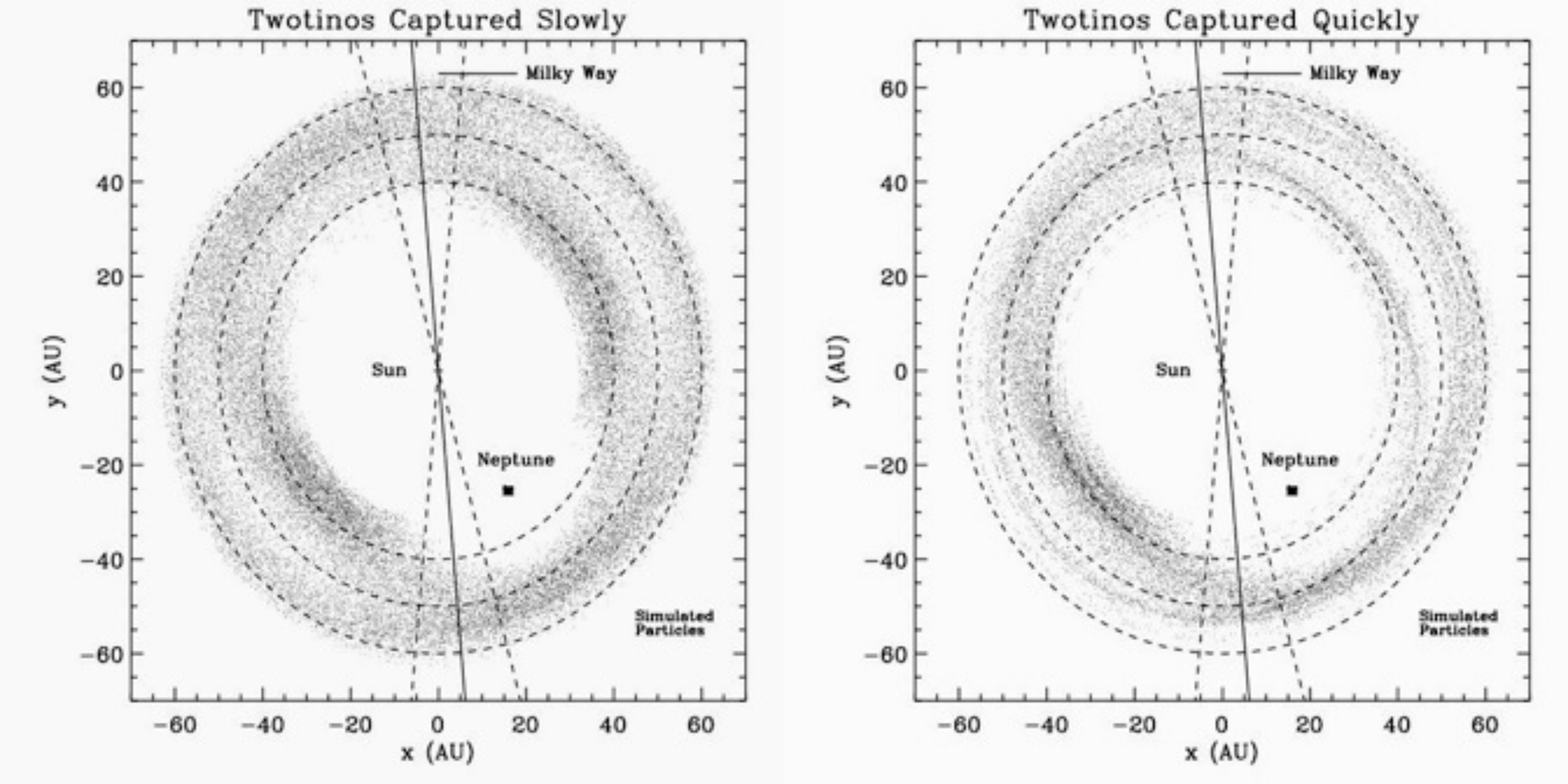


6-7 M_J



7-10 M_J

2:1 = Planetary Speedometer

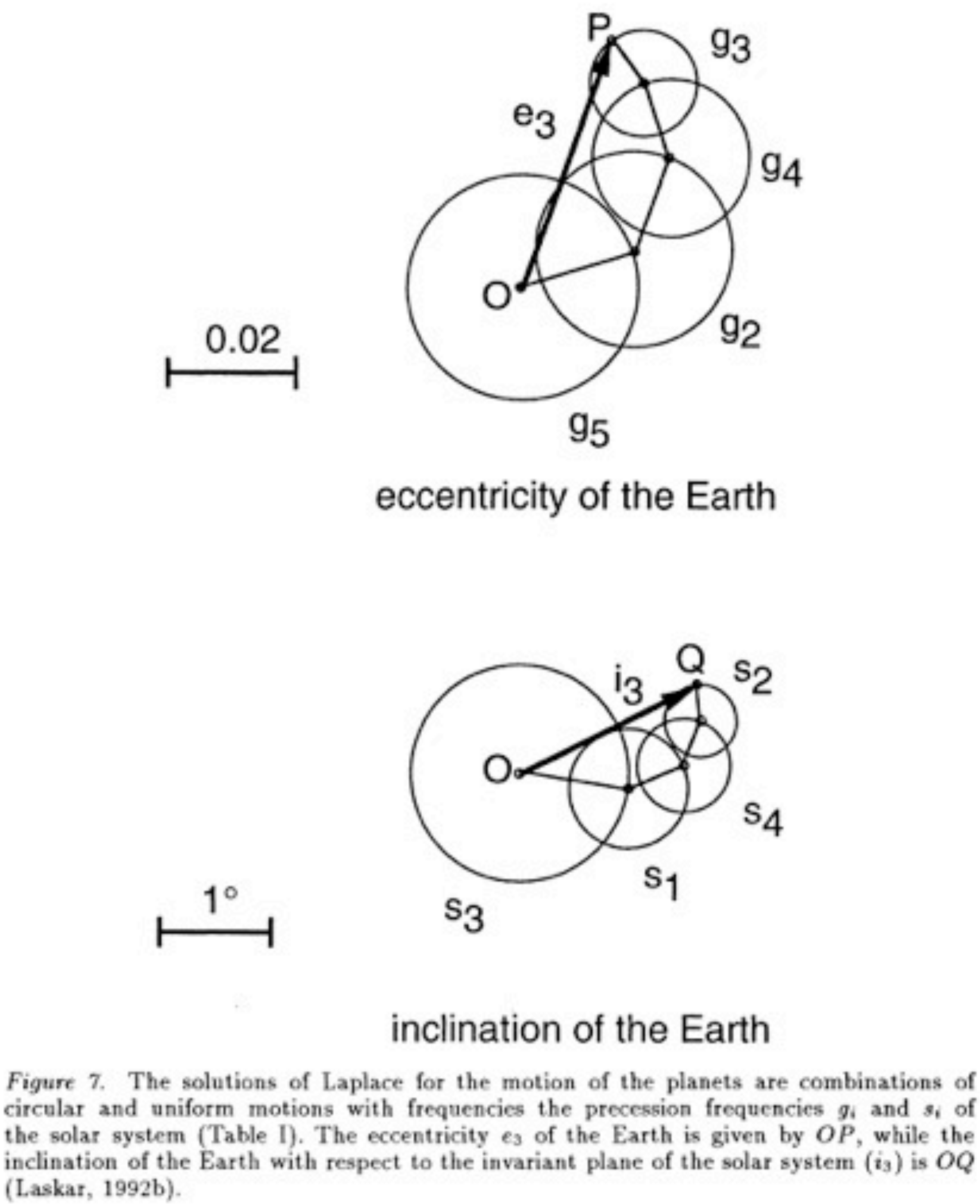


Twotinos Captured Slowly

Twotinos Captured Quickly

$t_{\mathrm{migrate}} \equiv a/(da/dt) \geq 10^{7}$ yr

$t_{\mathrm{migrate}} = 10^{6}$ yr



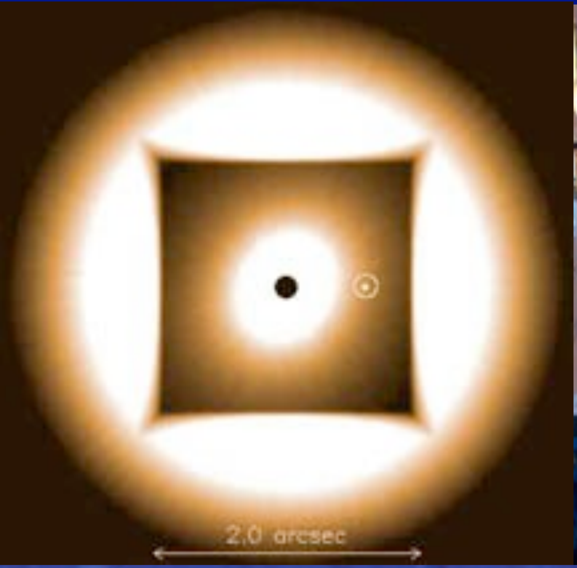
eccentricity of the Earth

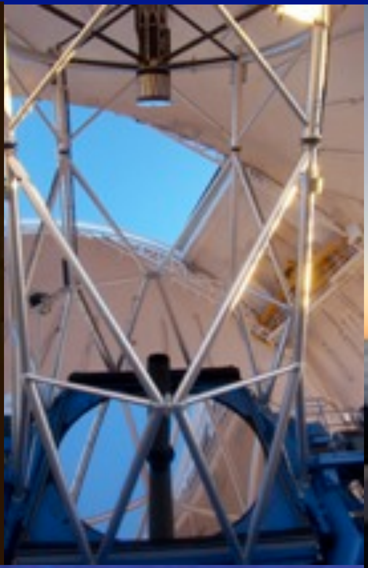
inclination of the Earth

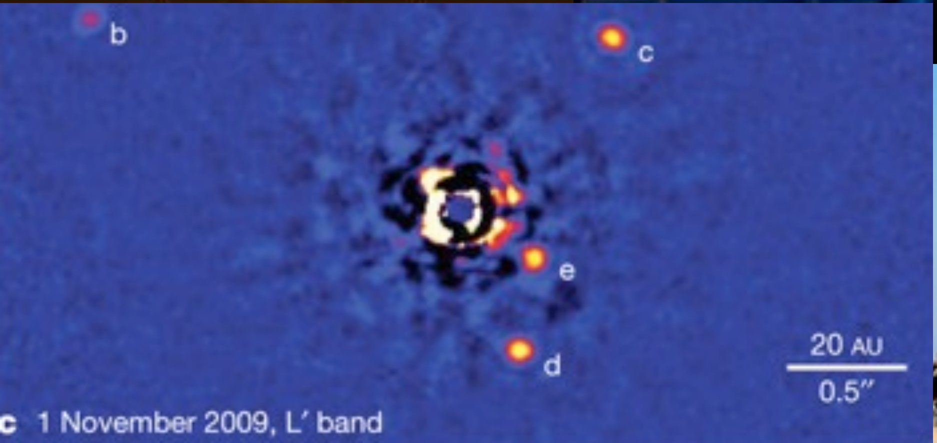
Figure 7. The solutions of Laplace for the motion of the planets are combinations of circular and uniform motions with frequencies the precession frequencies g_i and s_i of the solar system (Table I). The eccentricity e_3 of the Earth is given by OP, while the inclination of the Earth with respect to the invariant plane of the solar system (i_3) is OQ (Laskar, 1992b).

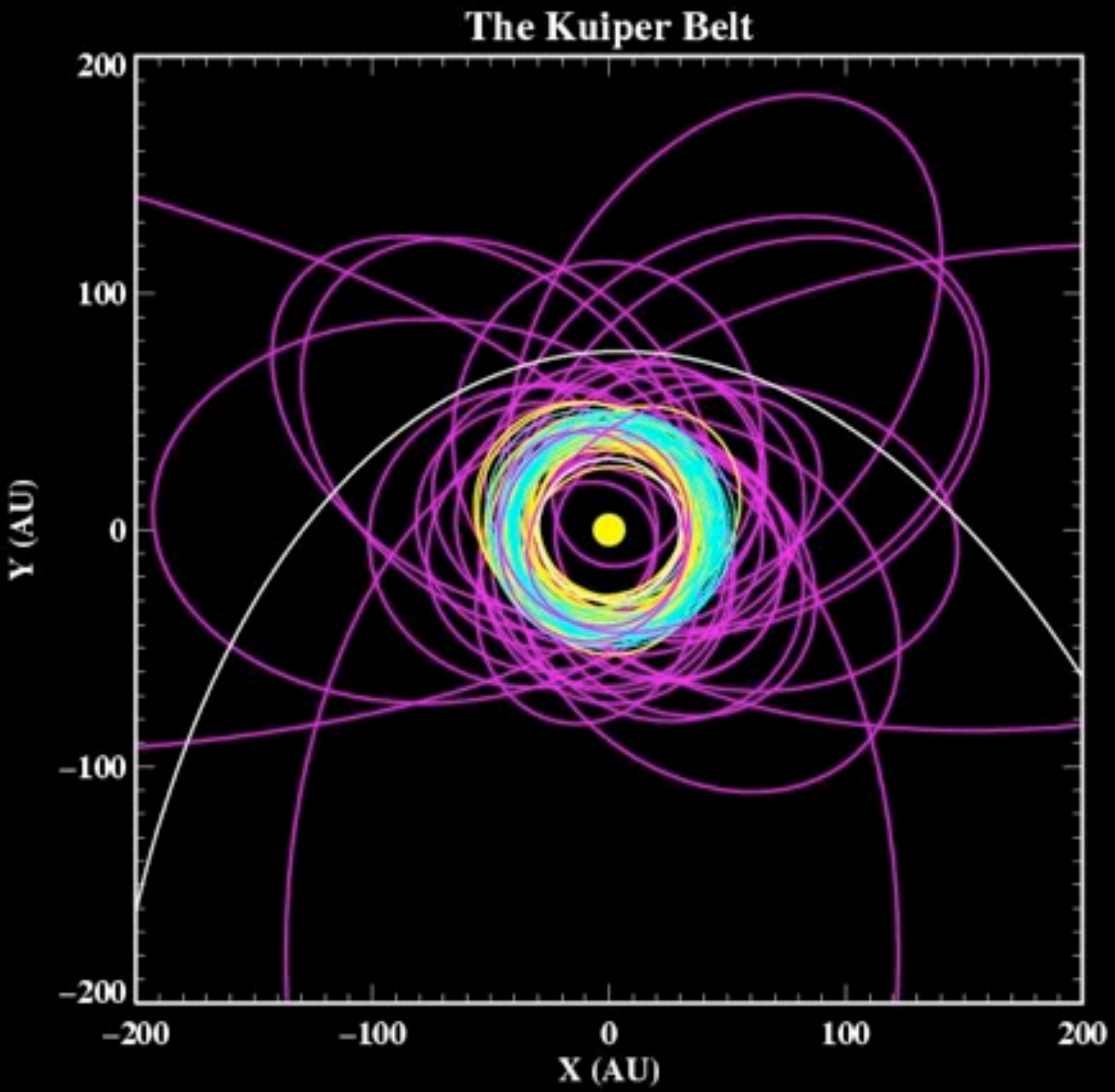
Gemini Planet Imager (GPI) 2012

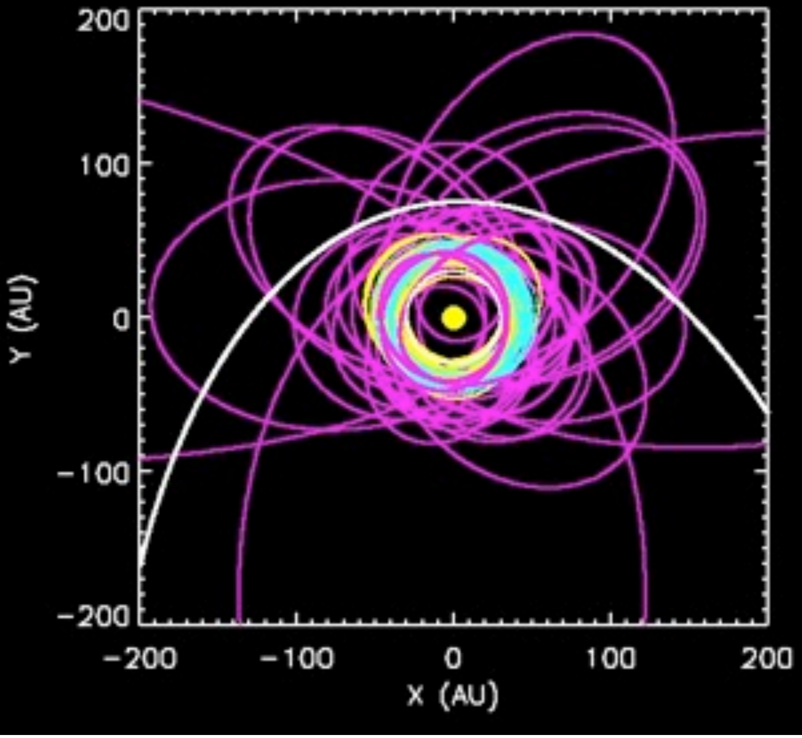
Pan-STARRS (once a week, mag 24) and LSST (once every few days, mag 24.5)

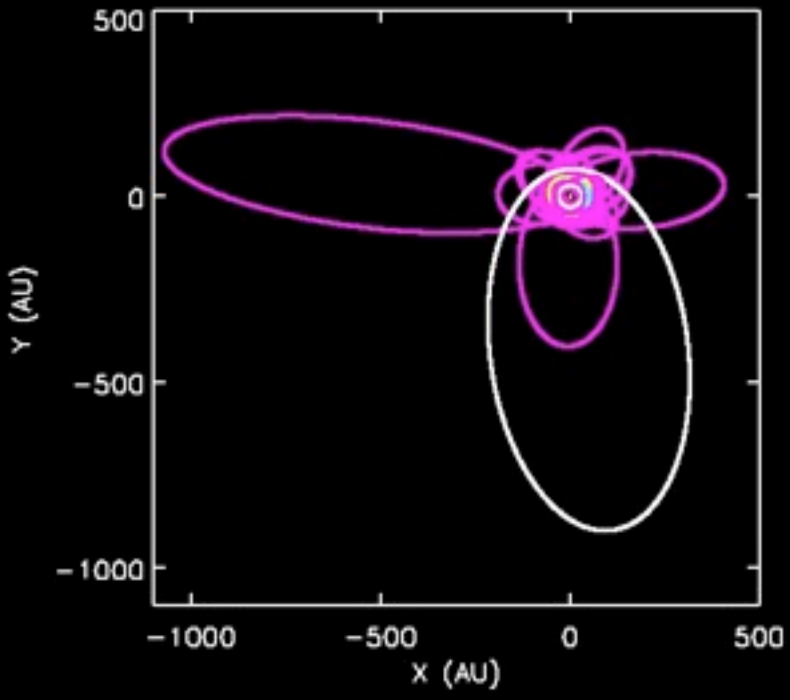


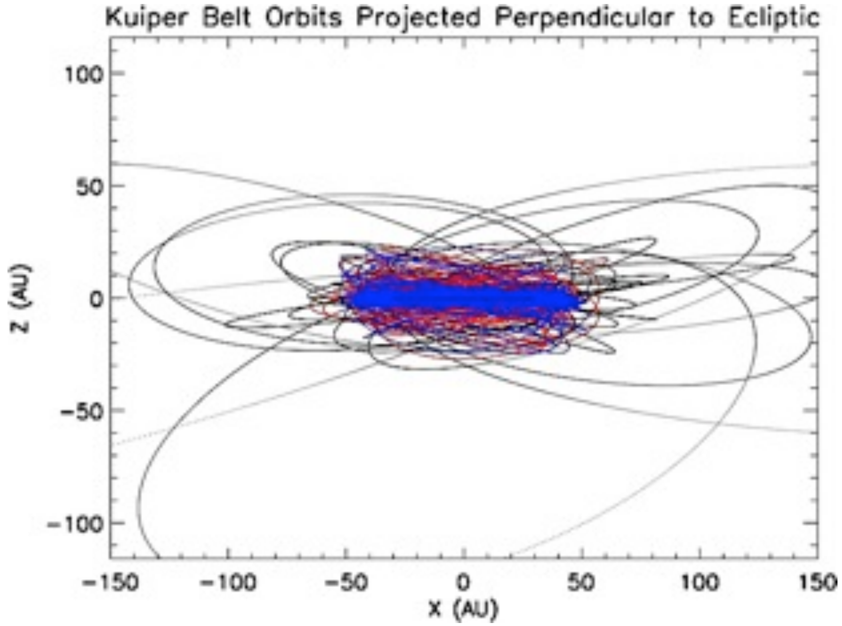





The Kuiper Belt
200
100
0
−100
−200
Y (AU)
−200
−100
0
100
200
X (AU)







$M \sim 0.1\ M_\oplus$

$$t_{\text{collision}} \sim \frac{1}{n\,\sigma\,v_{\text{rel}}} \sim 10^{12}\ \text{yr}$$

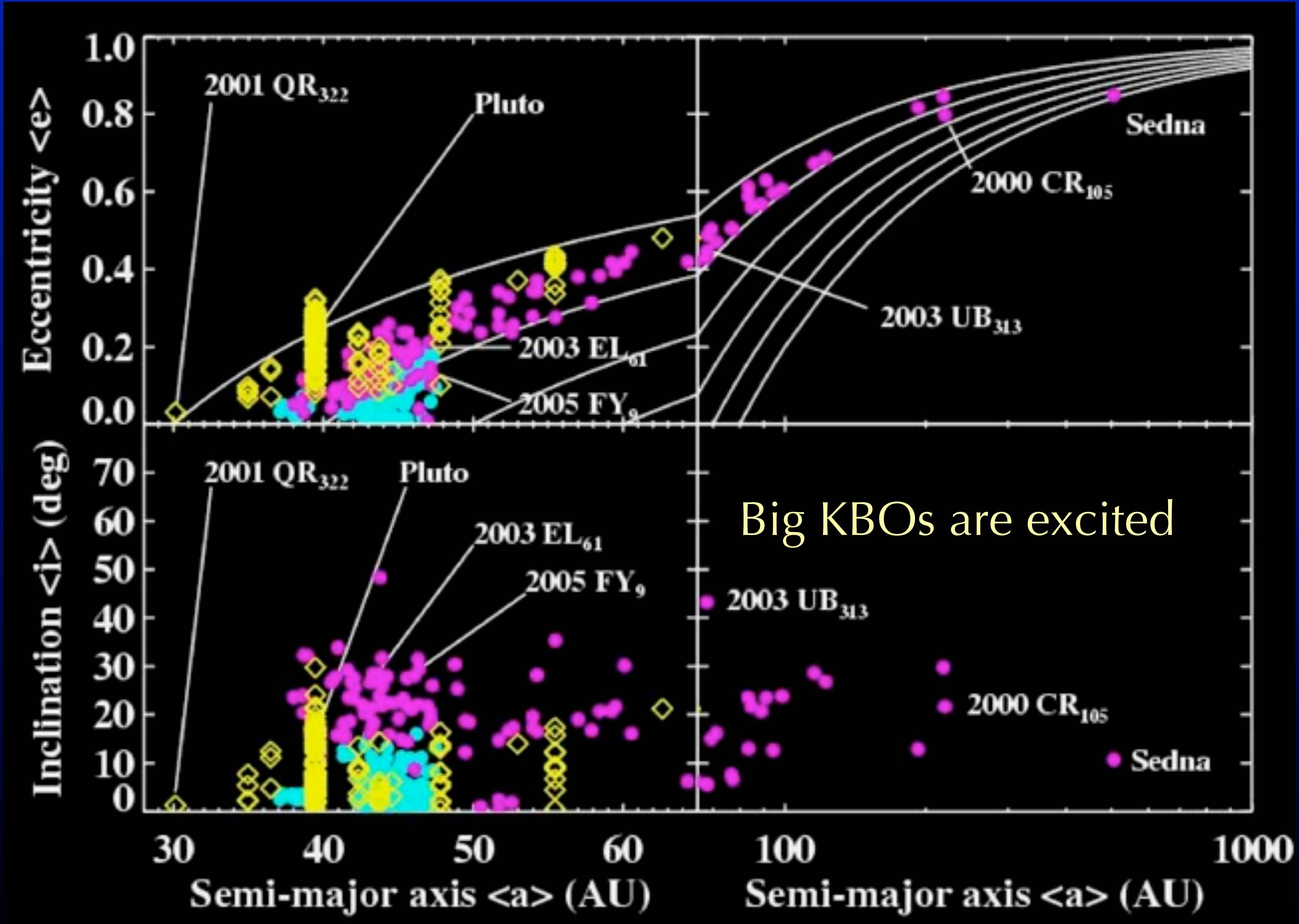
$3\ \text{AU}^{-3}$

$\pi\,(100\ \text{km})^2$

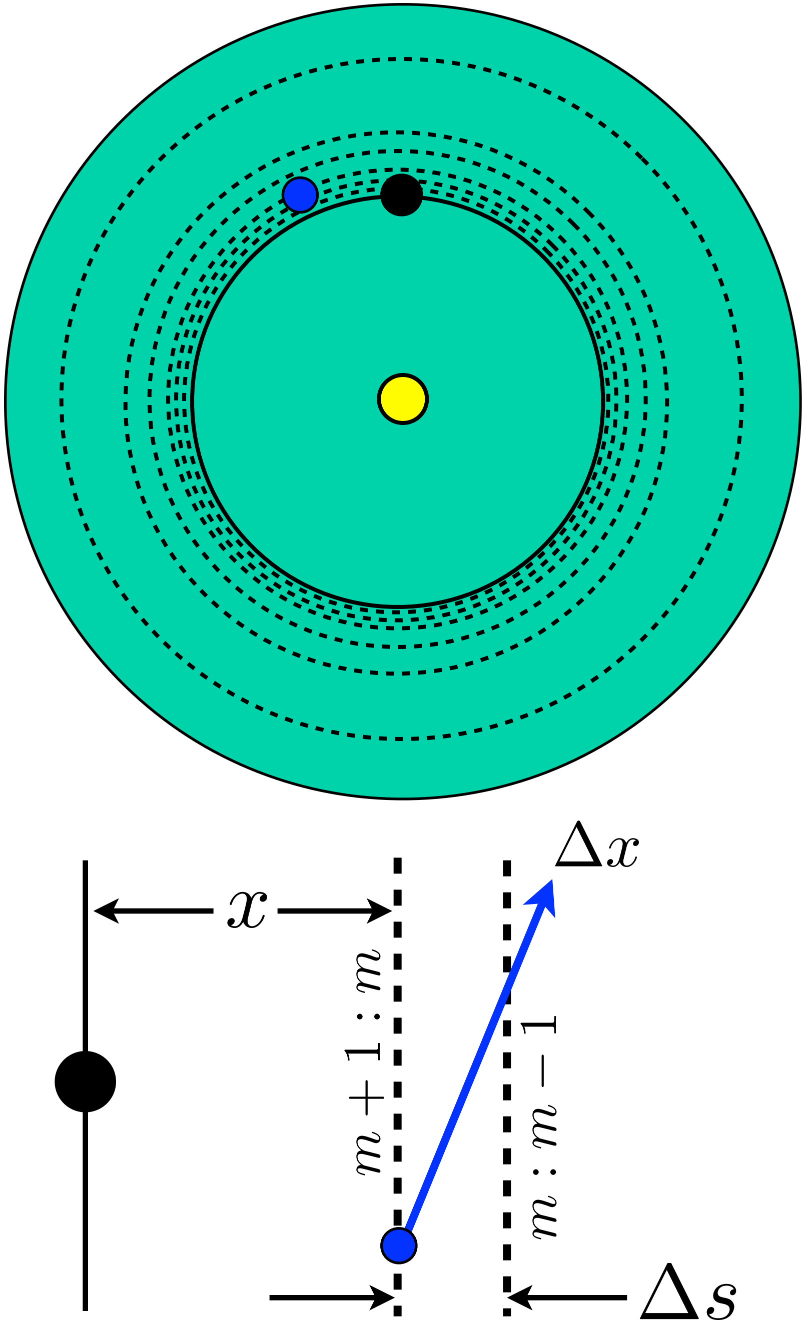
$1\ \text{km s}^{-1}$

KBOs = test particles

The Kuiper Belt: The Global View



Big KBOs are excited



Deriving the chaotic zone width

Resonance overlap

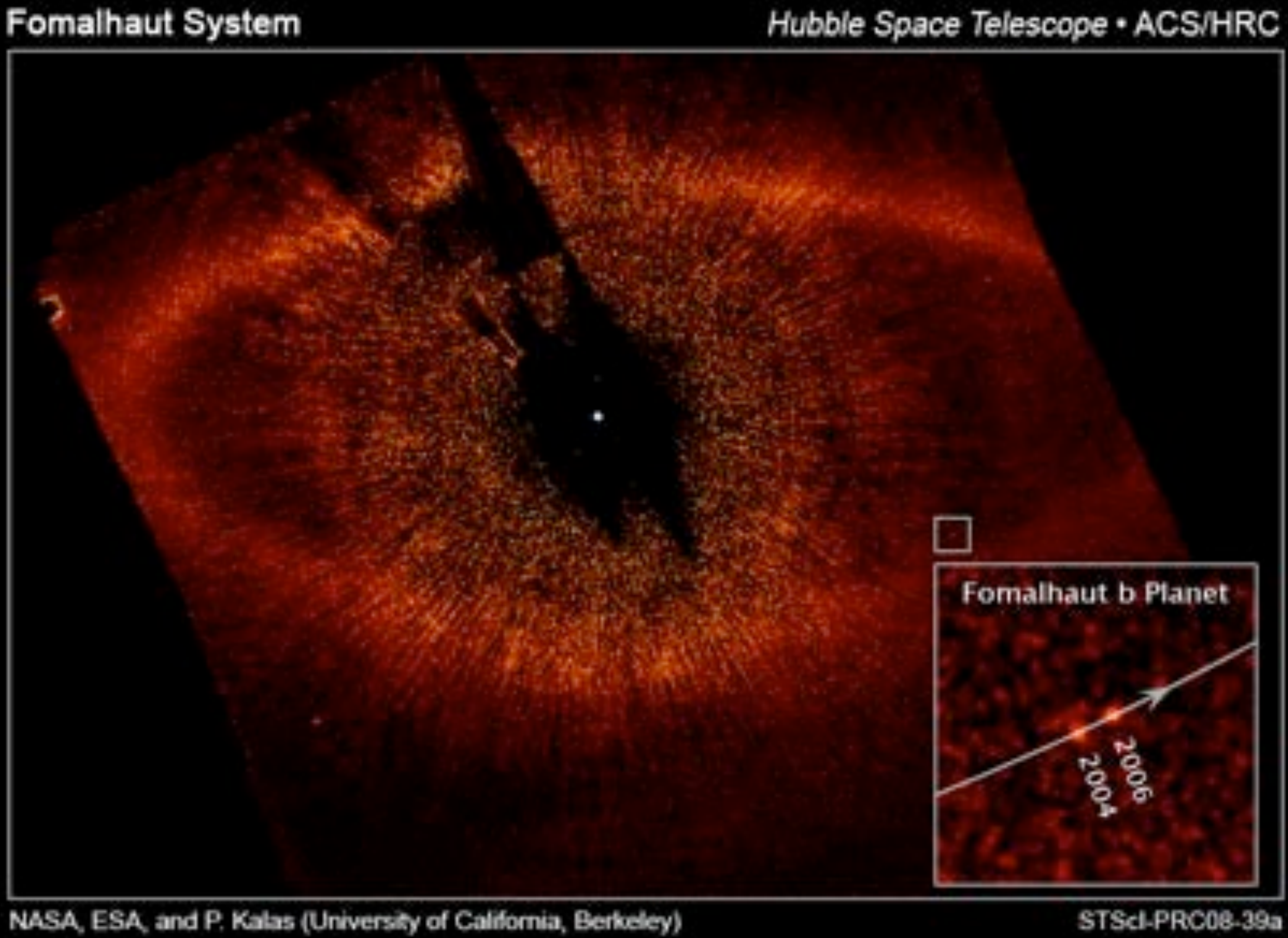
Resonance spacing

$$\frac{\Delta s}{a} \sim \left(\frac{x}{a}\right)^2$$

$$\text{if } \Delta x > \Delta s$$

$$\text{i.e., if } x < \left(\frac{M_p}{M_*}\right)^{2/7} a$$

then chaos



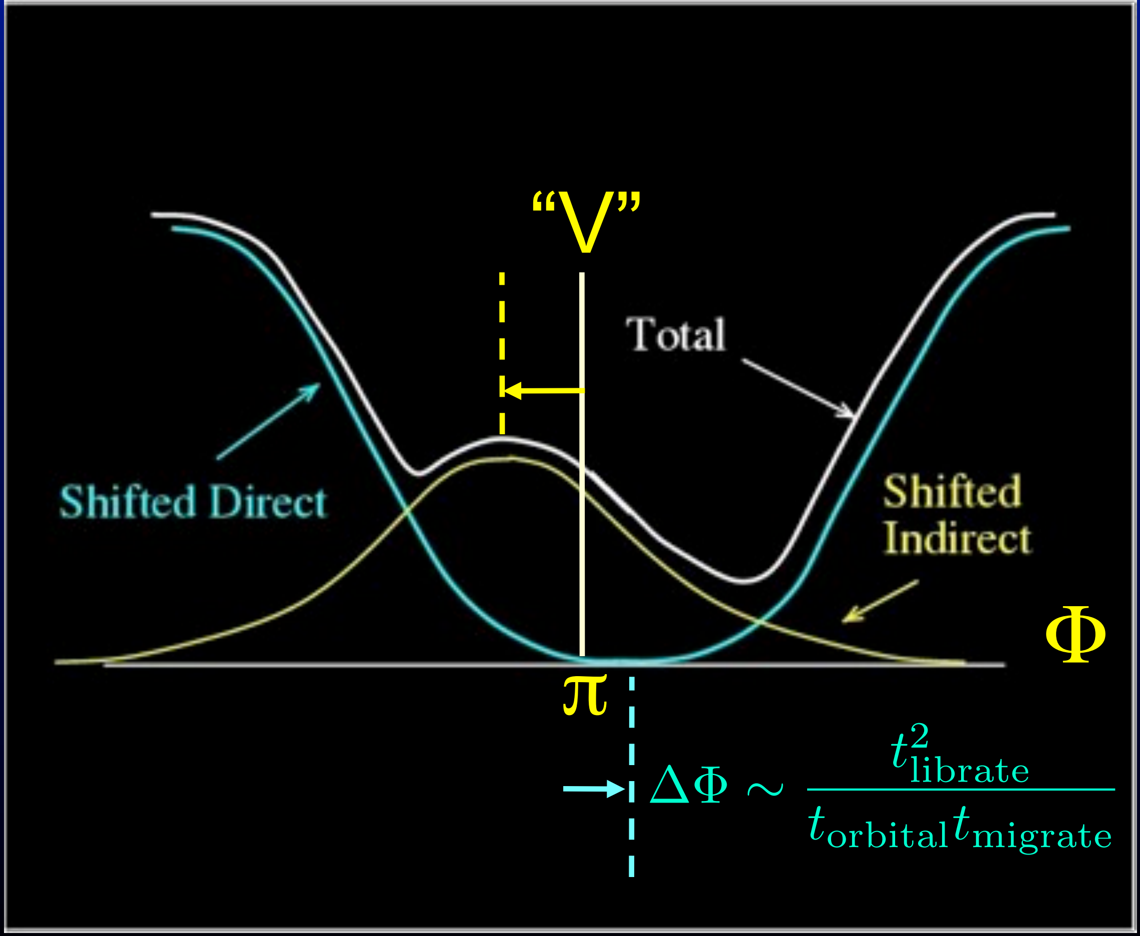
if $\omega_{\text{planet}} = \omega_{\text{belt}}$ (nested ellipses)

$$M_{\text{belt}} > M_{\text{parent bodies}} \sim 3M_{\oplus}$$

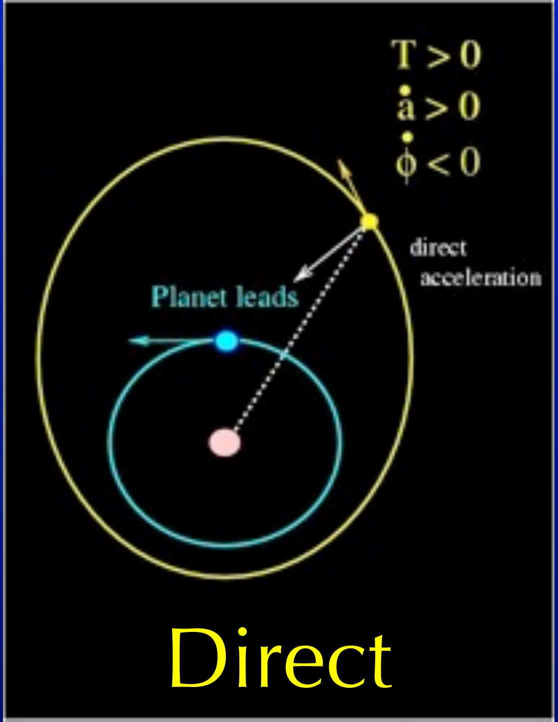
then
$$a_{\text{planet}} = 115\,\text{AU}$$
$$e_{\text{planet}} = 0.12$$
$$M_{\text{planet}} = 0.5M_{\text{J}}$$

Enough material for
gas giant core

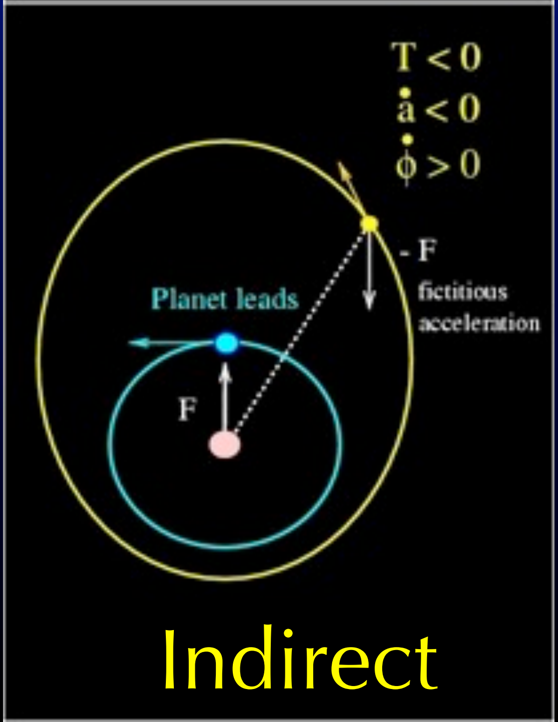
Asymmetric capture: Migration-shifted potentials



$$\Delta\Phi \sim \frac{t_{\mathrm{librate}}^2}{t_{\mathrm{orbital}} t_{\mathrm{migrate}}}$$



Direct



Indirect

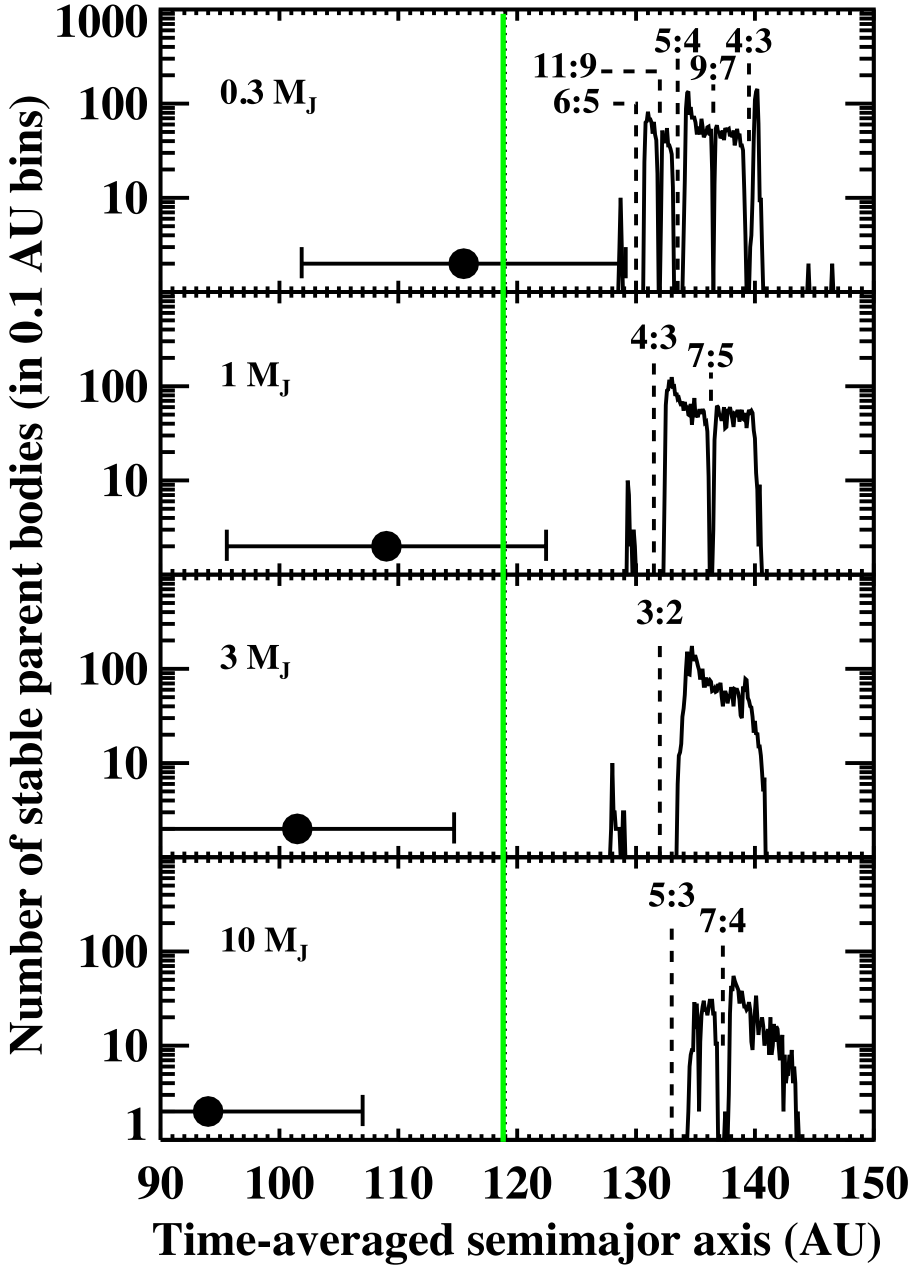
Results

- If $M_{\text{planet}} \uparrow$ then $a_{\text{planet}} \downarrow$

  Planet position too far from dust belt

  $\therefore\ M_{\text{planet}} < 3\,M_{\text{J}}$

- Planet also evacuates Kirkwood-type gaps



Chiang et al. 2008
Kalas et al. 2008

Results

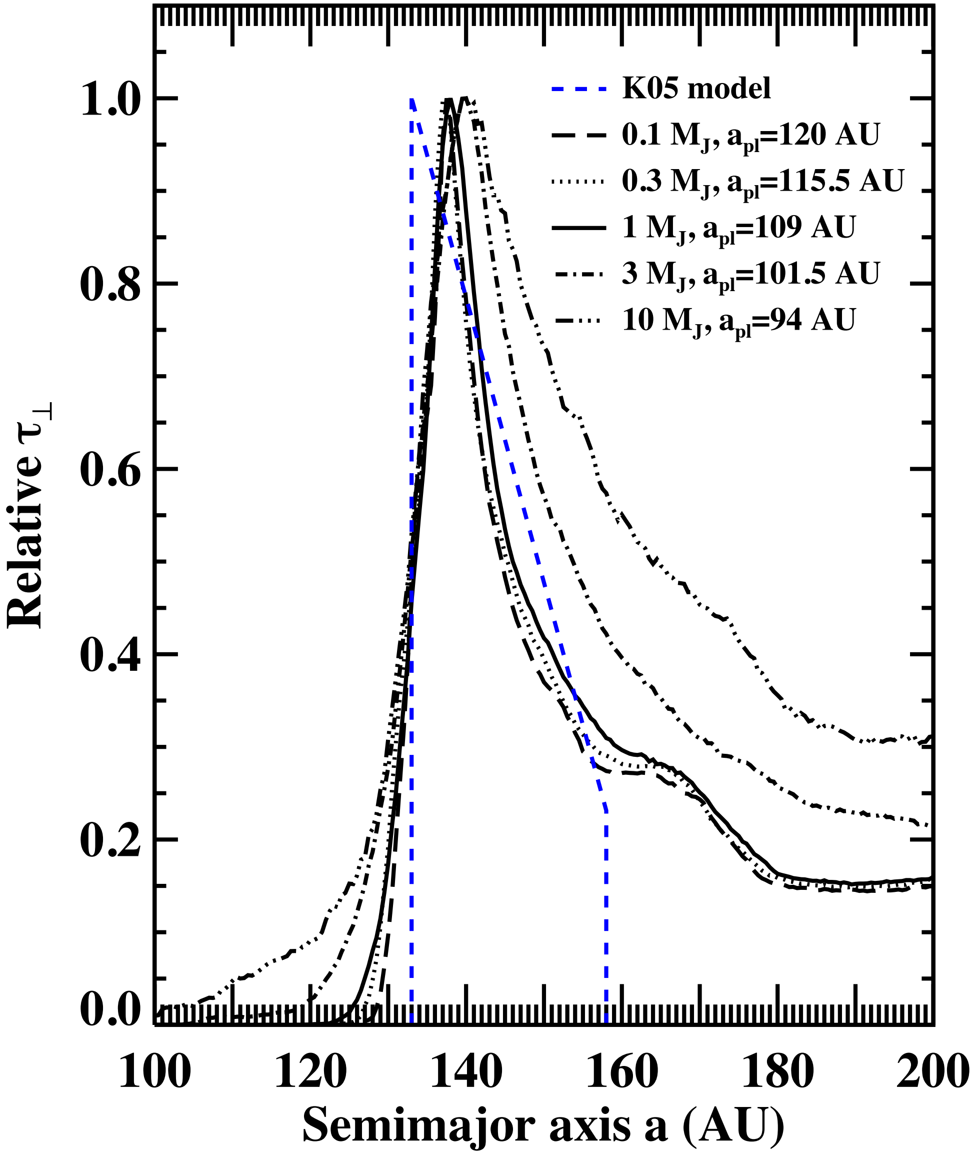
- If $M_{\mathrm{planet}} \uparrow$ then $e_{\mathrm{dust}} \uparrow$

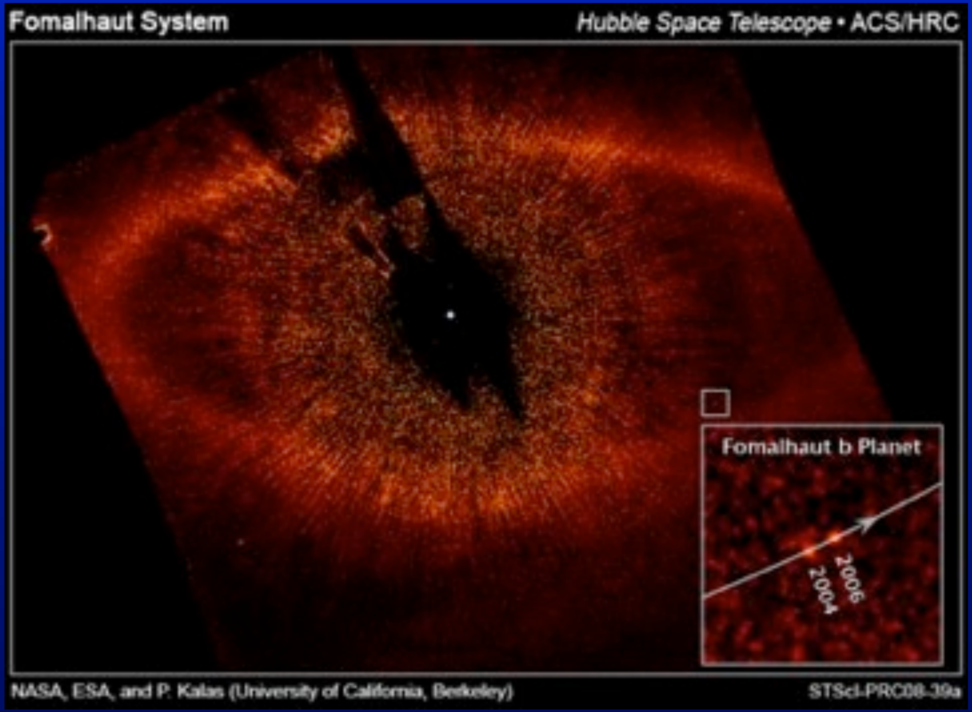
  Surface brightness profiles broaden too much

  $$\therefore\ M_{\mathrm{planet}} < 3\,M_{\mathrm{J}}$$

- $M_{\mathrm{belt}} > M_{\mathrm{parent\ bodies}} \sim 3M_{\oplus}$

  Enough material for gas giant core





NASA, ESA, and P. Kalas (University of California, Berkeley)

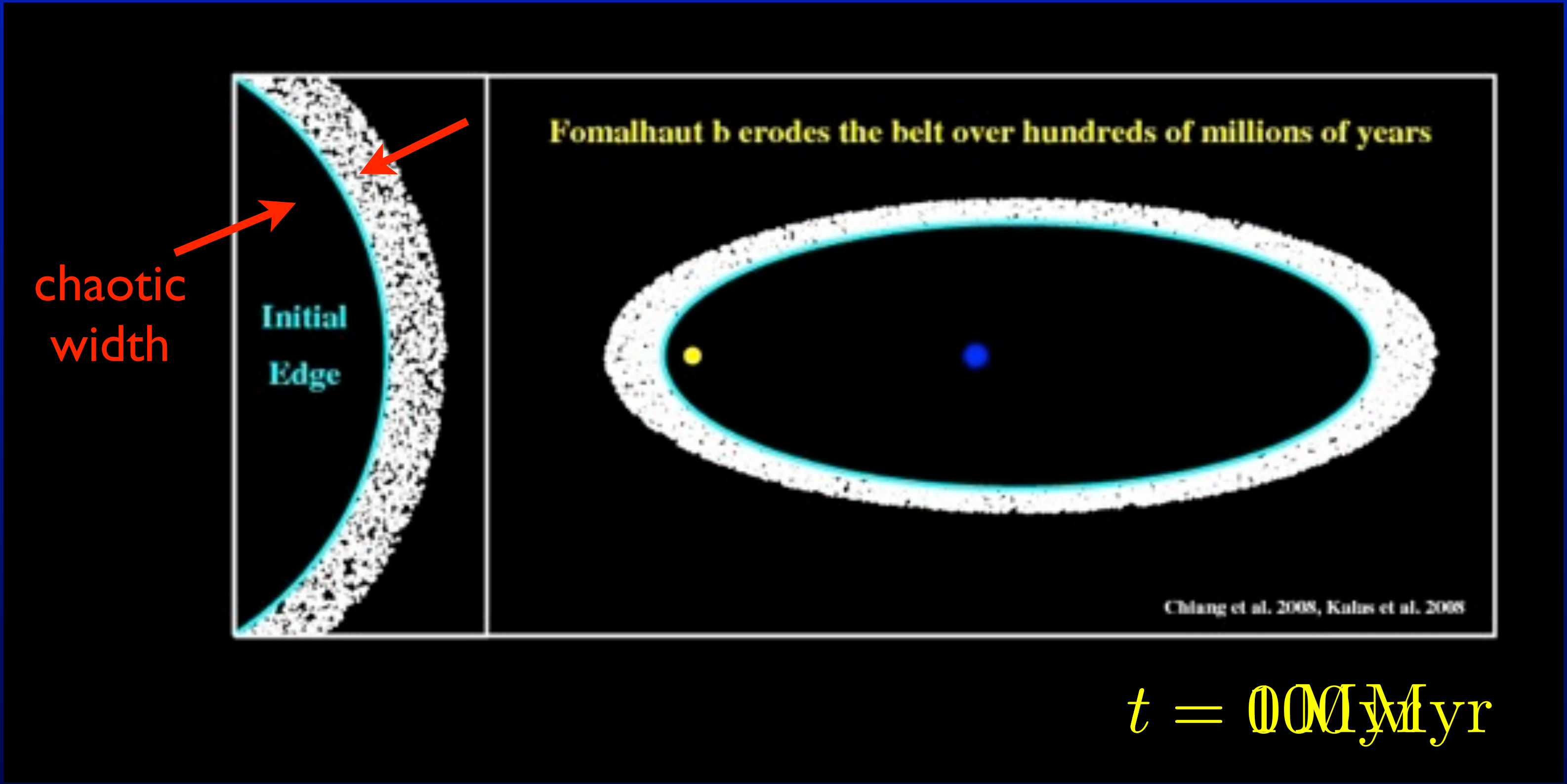
STScI-PRC08-39a

$$\text{if } \omega_{\text{planet}} = \omega_{\text{belt}} \text{ (nested ellipses)}$$

$$\text{then} \quad \begin{aligned} a_{\text{planet}} &= 115\text{ AU} \\ e_{\text{planet}} &= 0.12 \\ M_{\text{planet}} &= 0.5 M_{\text{J}} \end{aligned}$$

- The Kuiper belt comprises tens of thousands of icy, rocky objects having sizes greater than 100 km
- Many KBOs occupy highly eccentric and inclined orbits that imply a violent past
- Pluto and other Resonant KBOs share special gravitational relationships with Neptune
- Extrasolar debris disks are nascent Kuiper belts
- Belts are gravitationally sculpted by planets

Fomalhaut b: Planet-Debris Disk Interaction



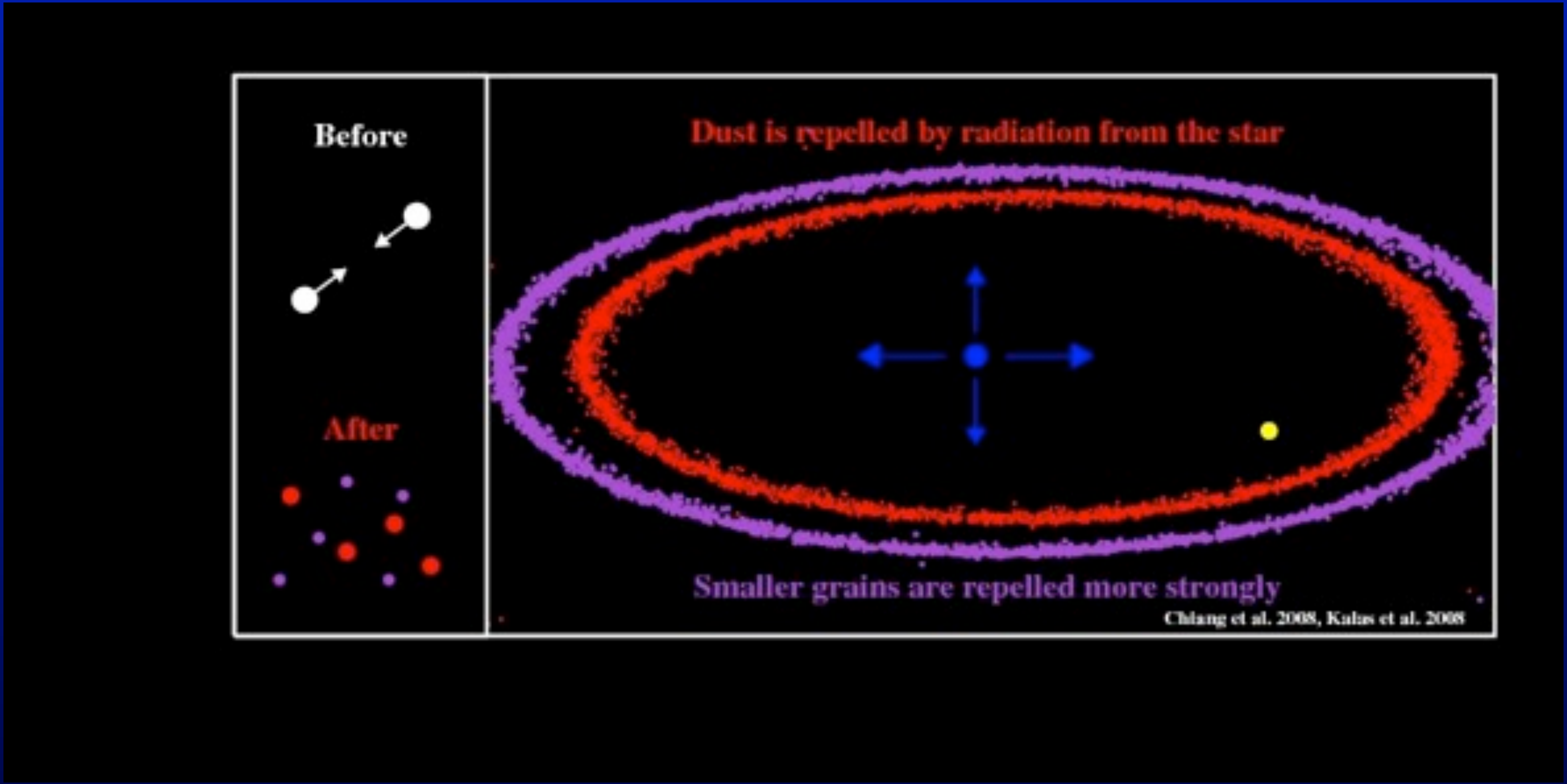
chaotic width

Chiang et al. 2008, Kalas et al. 2008

$t = 100\ \mathrm{Myr}$

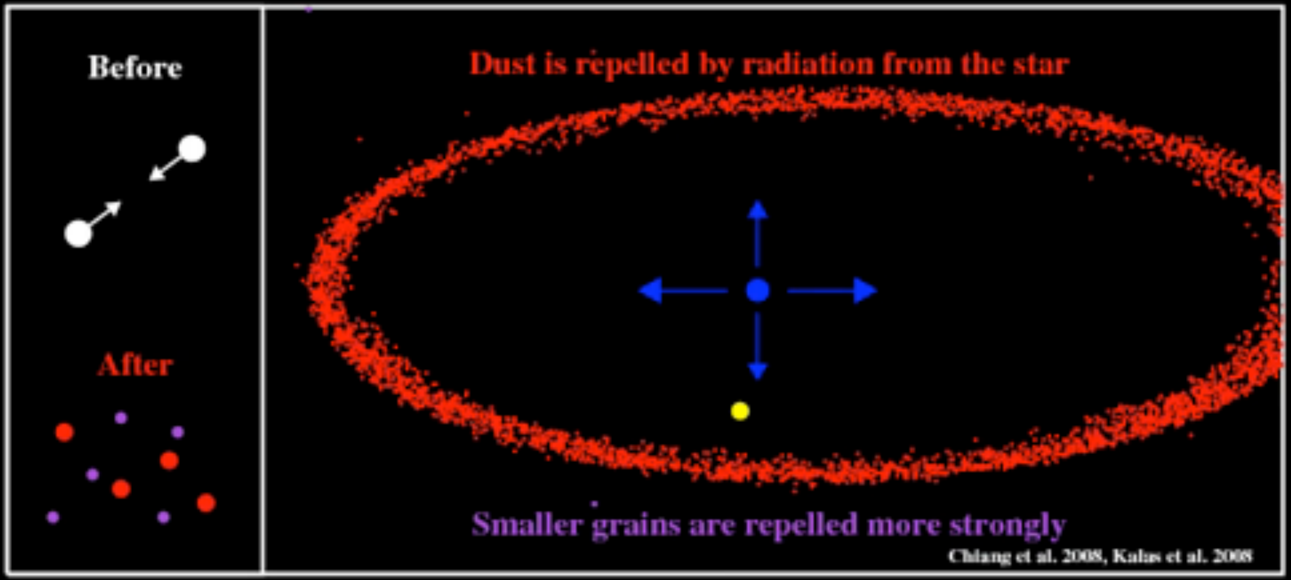
Step 1: Screen parent bodies for gravitational stability

$$t_{\mathrm{age}} \sim 10^8\ \mathrm{yr}$$



Chiang et al. 2008, Kalas et al. 2008

Step 2: Replace parent bodies with dust grains



Before

After

Dust is repelled by radiation from the star

Smaller grains are repelled more strongly

Chiang et al. 2008, Kalas et al. 2008

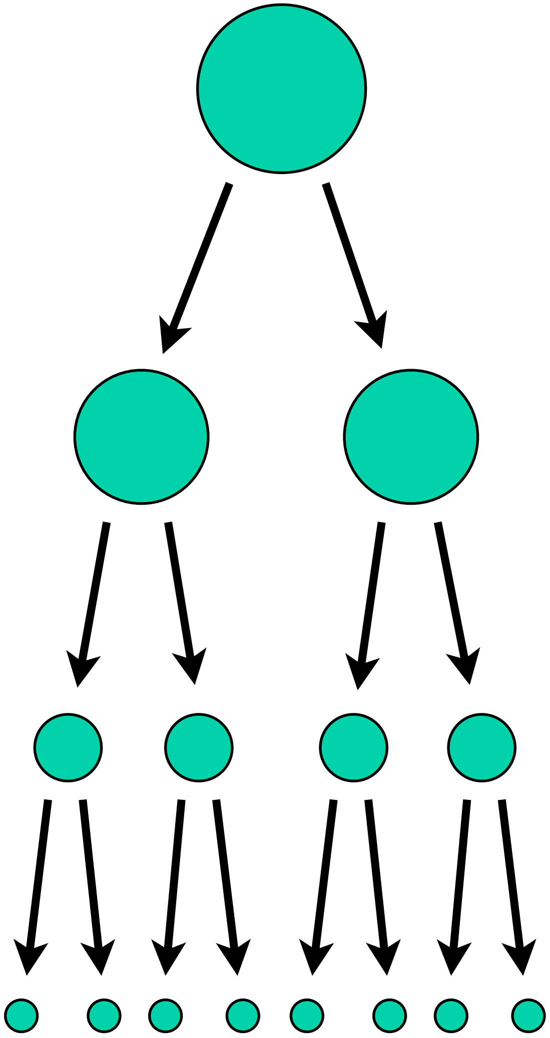
$$\Delta t = t_{\rm collision} = 0.1 \ {\rm Myr}$$

Step 3: Integrate dust grains with radiative force
for collisional lifetime

$$t_{\rm collision} \sim t_{\rm orb}/\tau \sim 0.1 \ {\rm Myr}$$

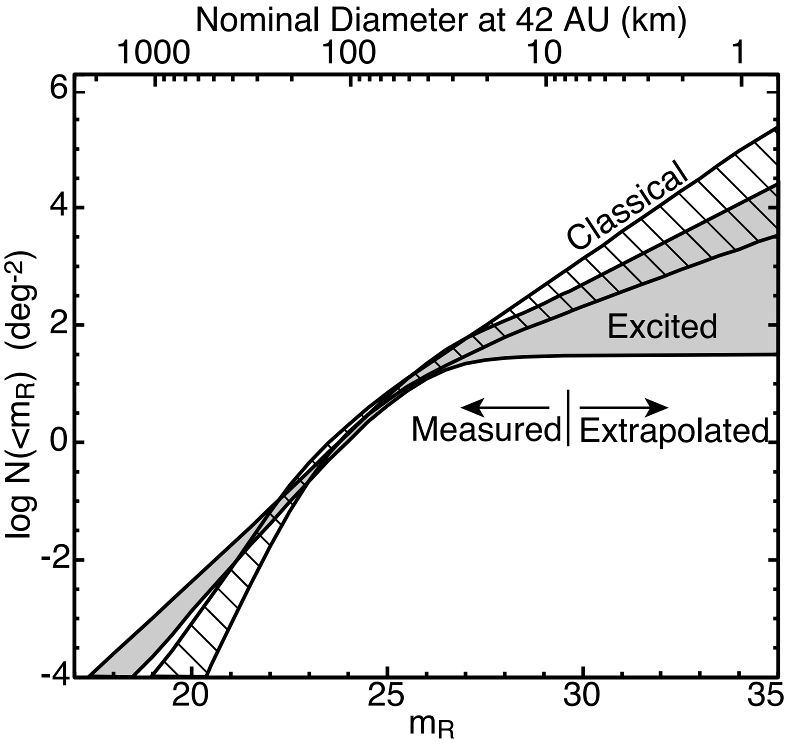
Collisional Cascade

Parent bodies collide once in system age



Most visible grains are just large enough to avoid stellar blow-out

Distribution of sizes of Kuiper belt objects



Results

- If $M_{\rm planet} \uparrow$ then $a_{\rm planet} \downarrow$

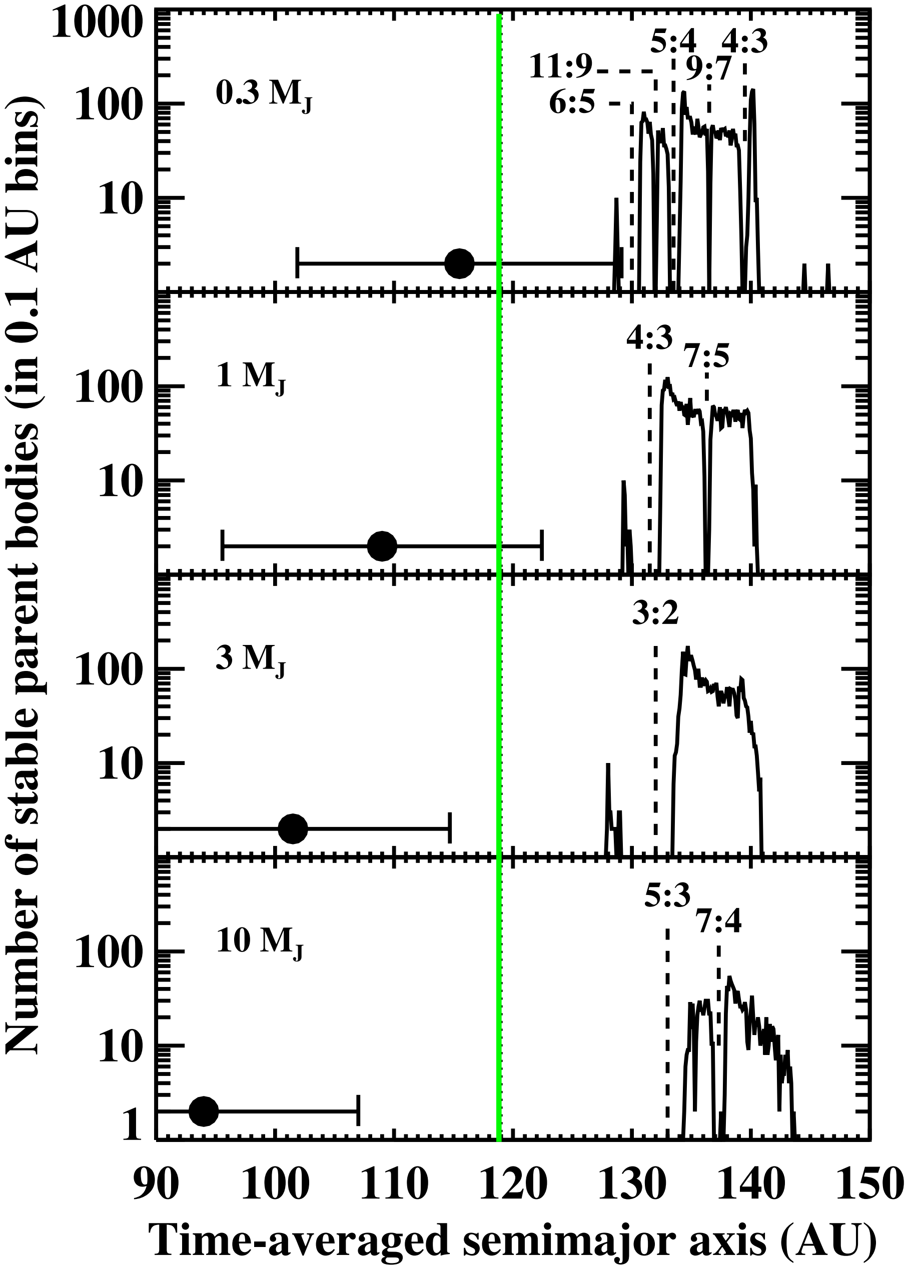
  Planet position too far from dust belt

  $\therefore\ M_{\rm planet} < 3\,M_{\rm J}$

- Planet also evacuates Kirkwood-type gaps

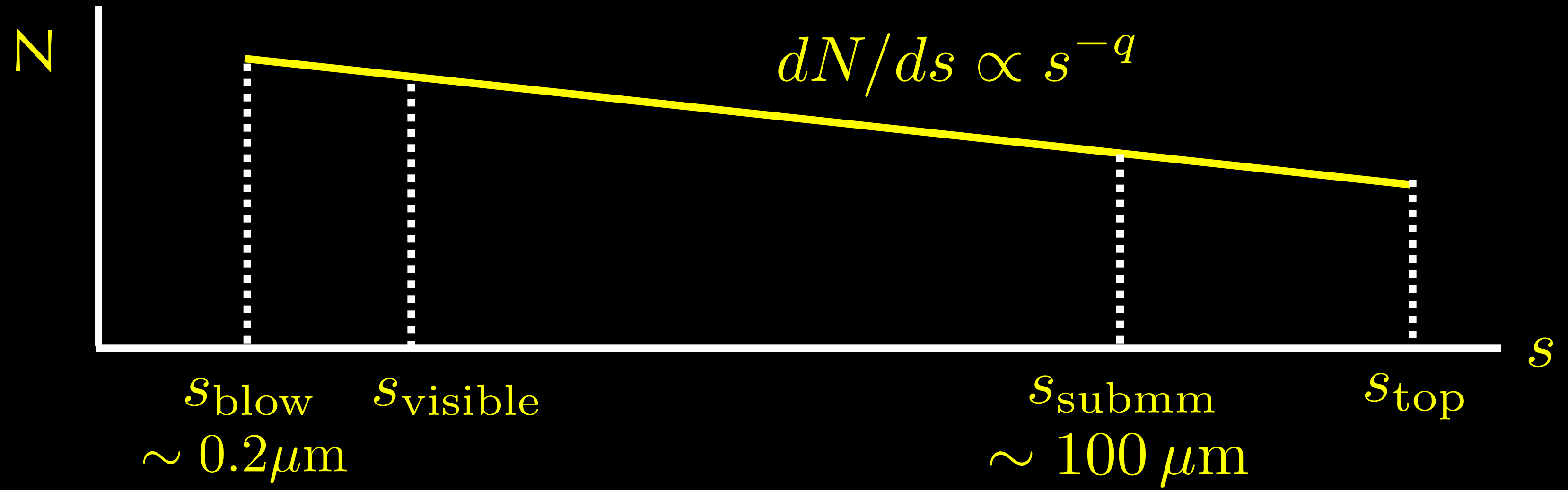
- $M_{\rm belt} > M_{\rm parent\ bodies} \sim 3 M_\oplus$

  Enough material for gas giant core



Chiang et al. 2008
Kalas et al. 2008

Measuring Debris Disk Masses



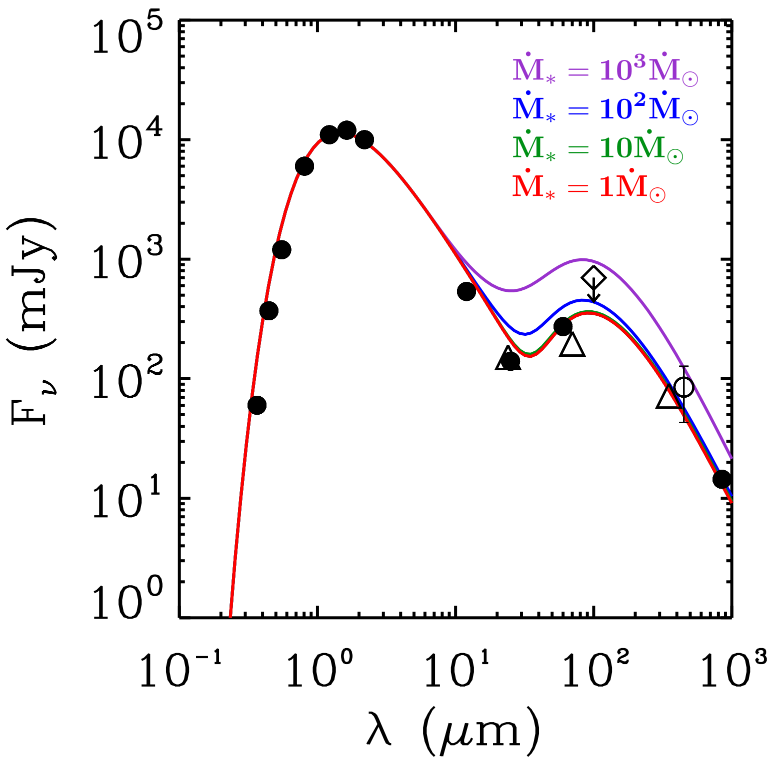
850 μm flux consistent w/

$M_{\rm SED}(s_{\rm top}) \sim 0.01 M_\oplus$

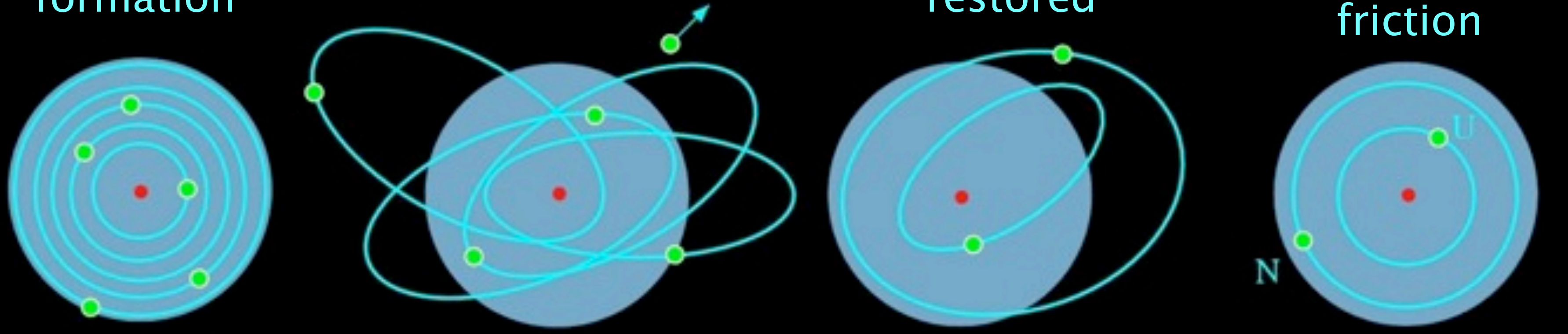
$q = 7/2$(Dohnanyi)

$s_{\rm top} \sim 10$ cm

[from $t_{\rm collision}(s_{\rm top}) \sim t_{\rm age}$]



Packed planetary systems



Packed formation

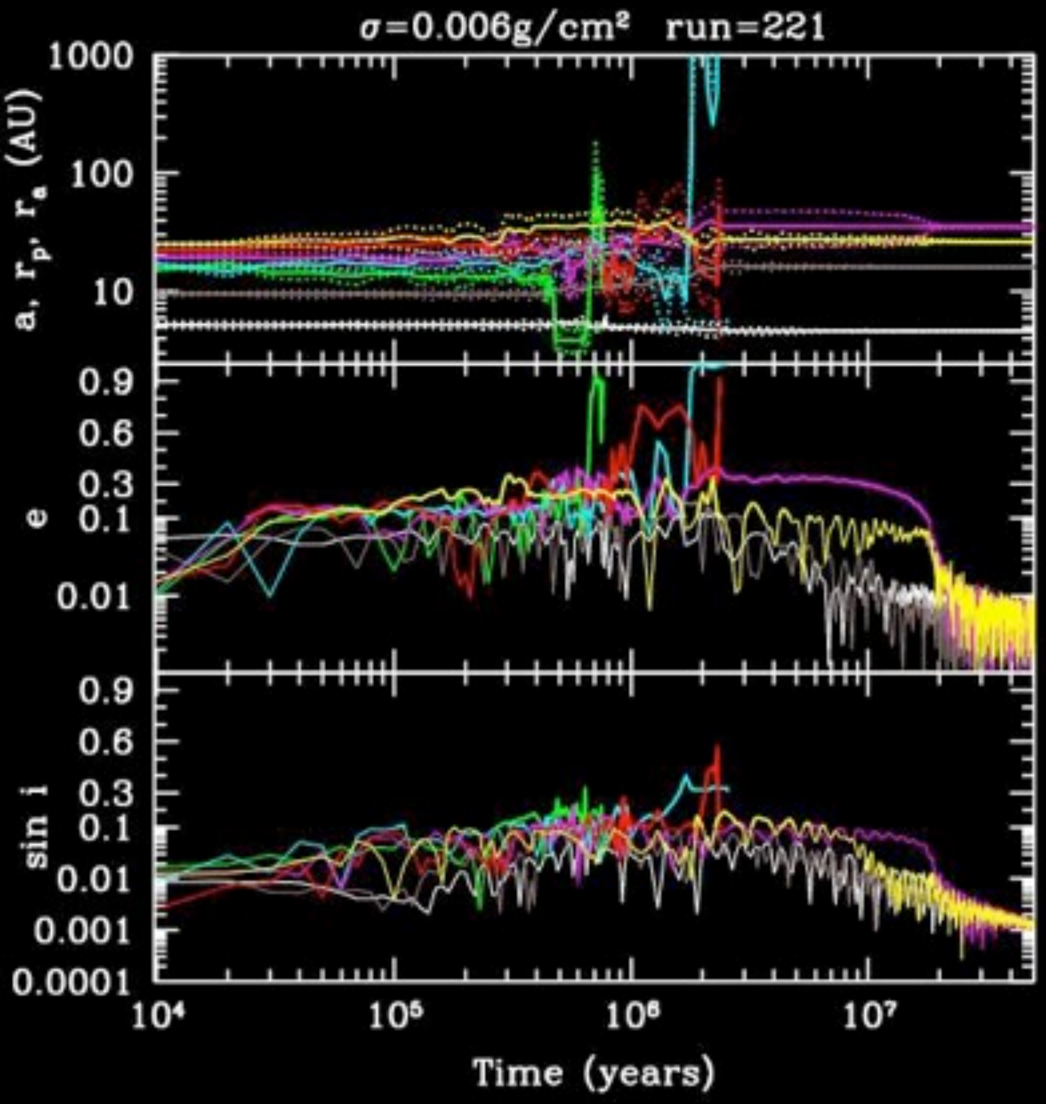
Instability and ejection

Stability restored

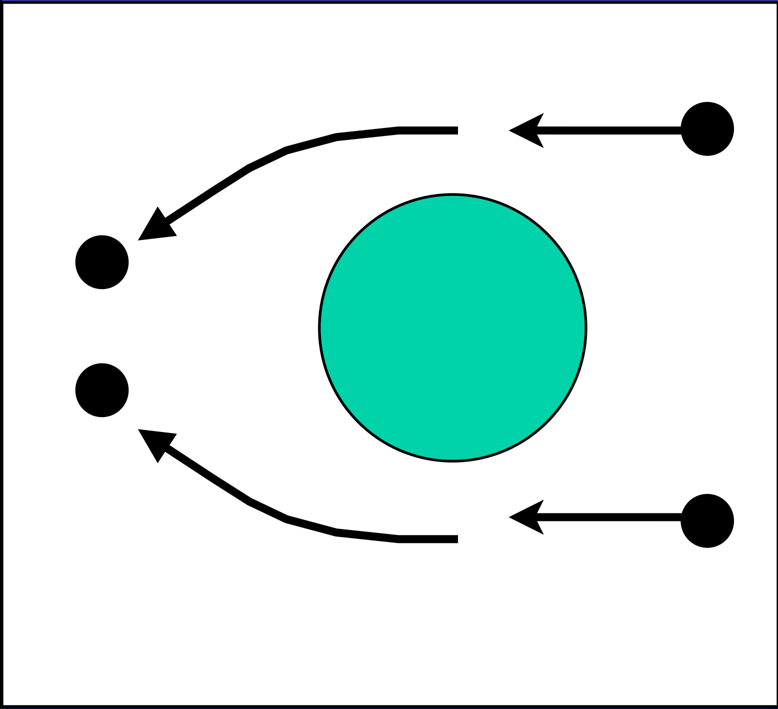
Regularization by dynamical friction

occurs when planets outweigh parent disk

Signature recorded in Kuiper belt

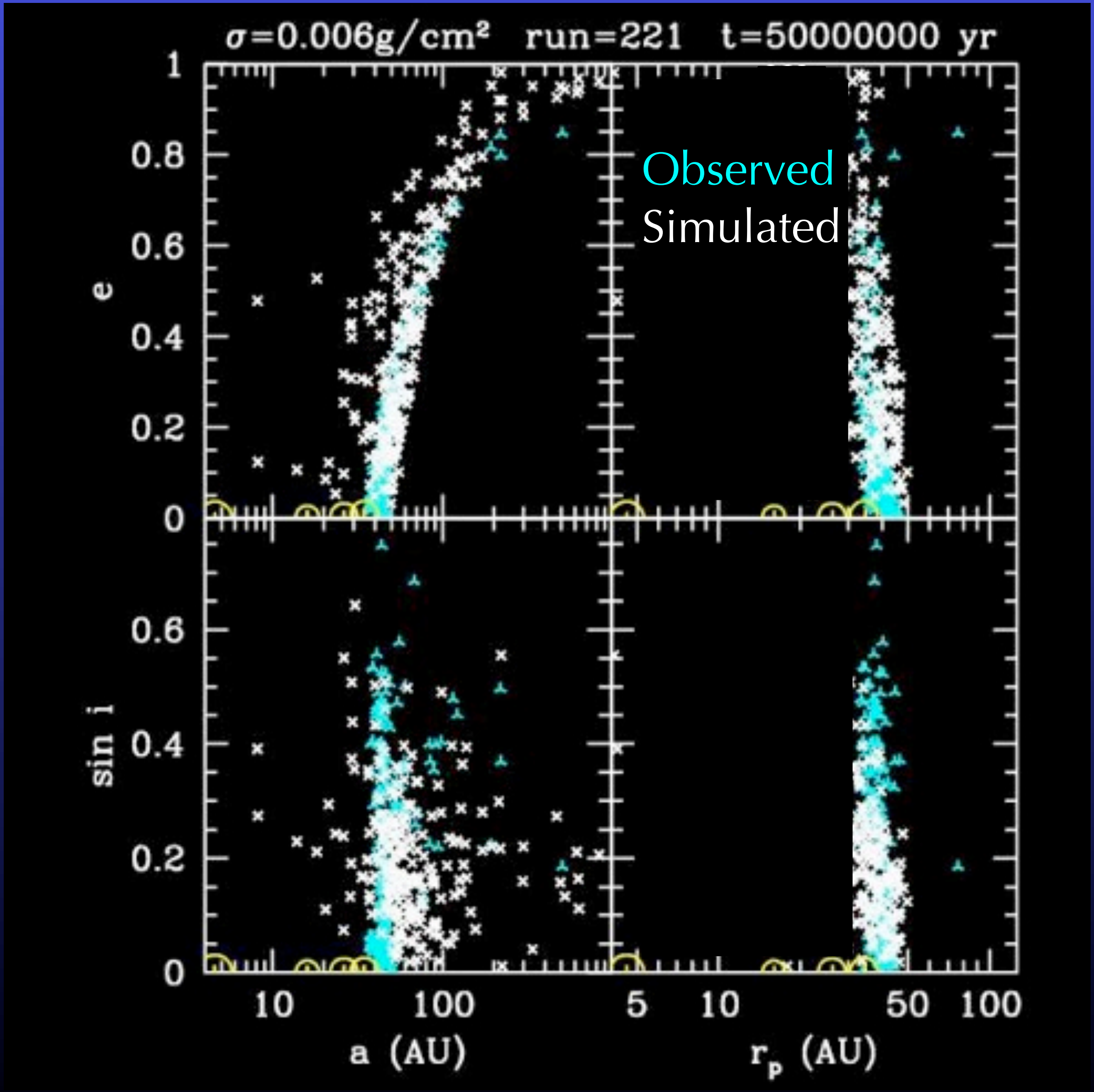


Dynamical friction:
Small bodies slow
big bodies



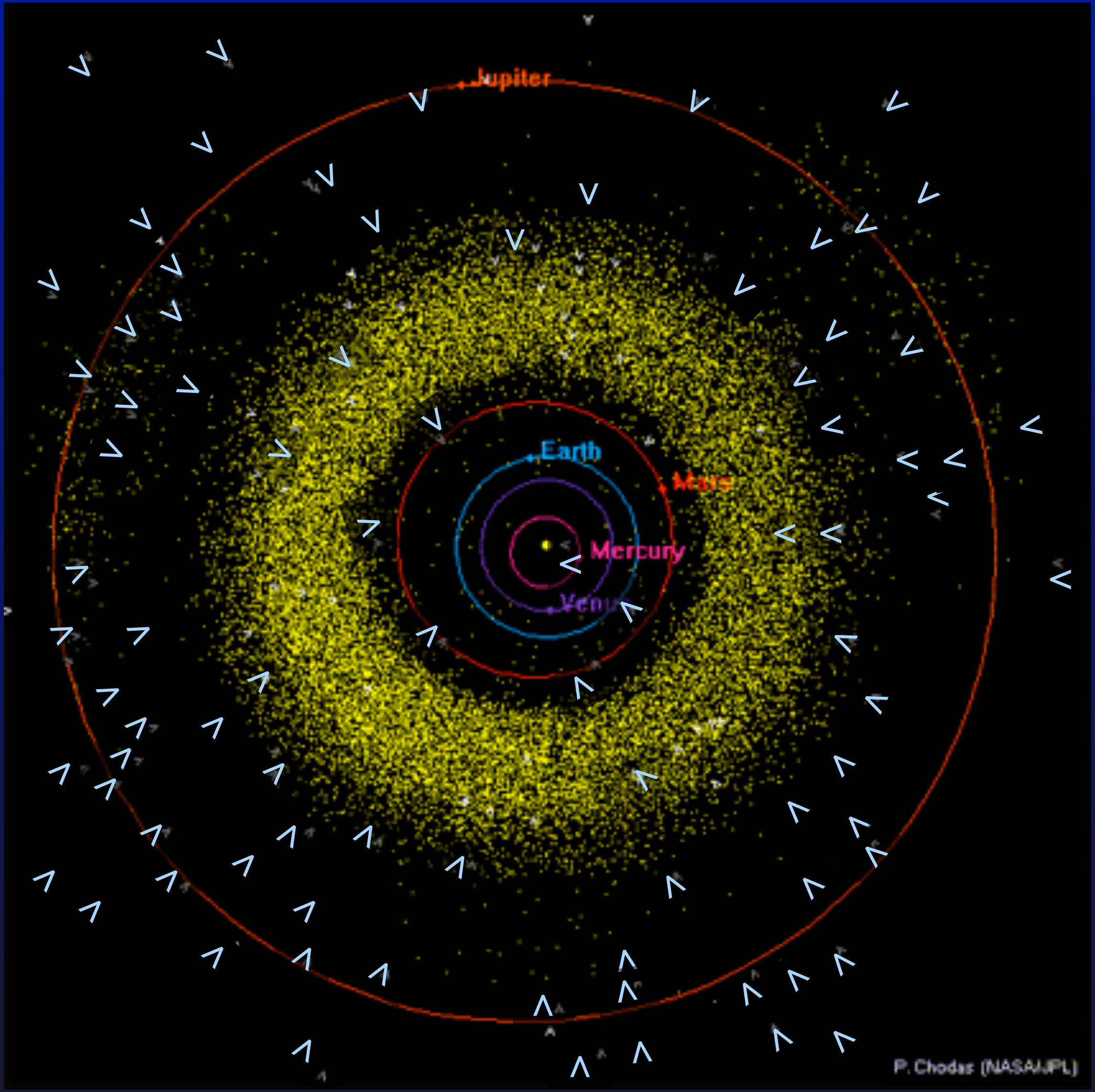
Numerical simulations of packed planetary systems

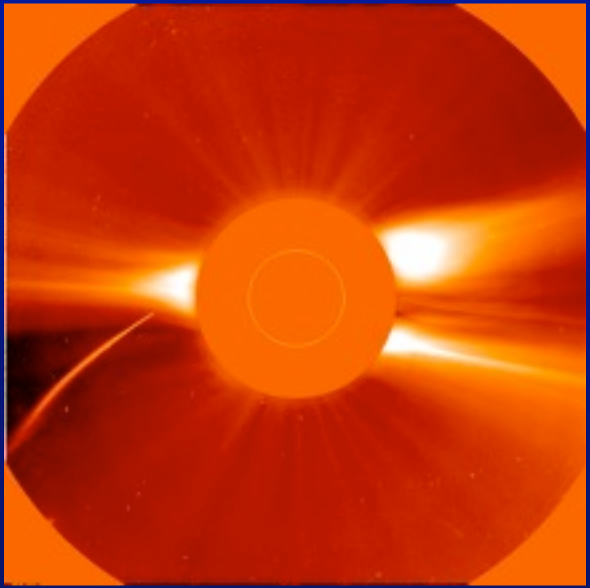
Solar system-like outcomes emerge from chaos



Stirring of KBOs by Rogue Ice Giants

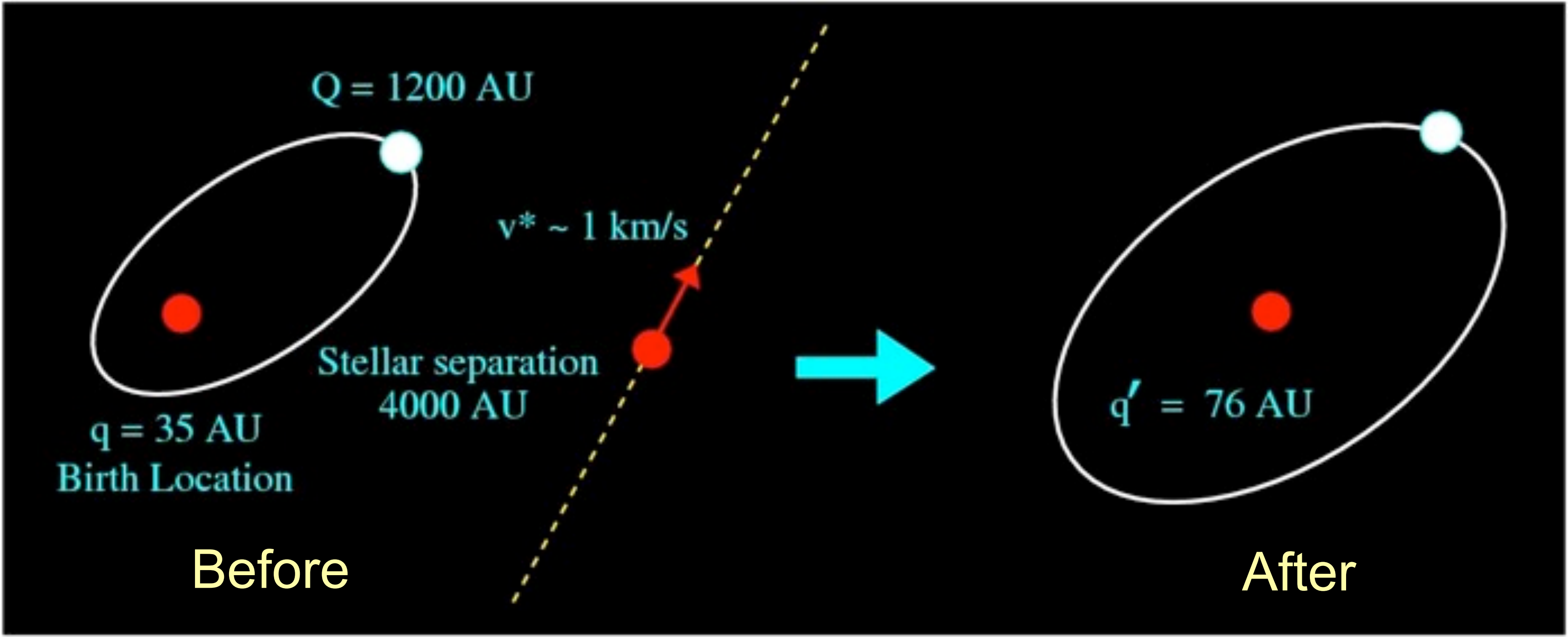
Short-Period Comets





Raising Sedna's Peri by Stellar Encounters

Praesepe cluster M44

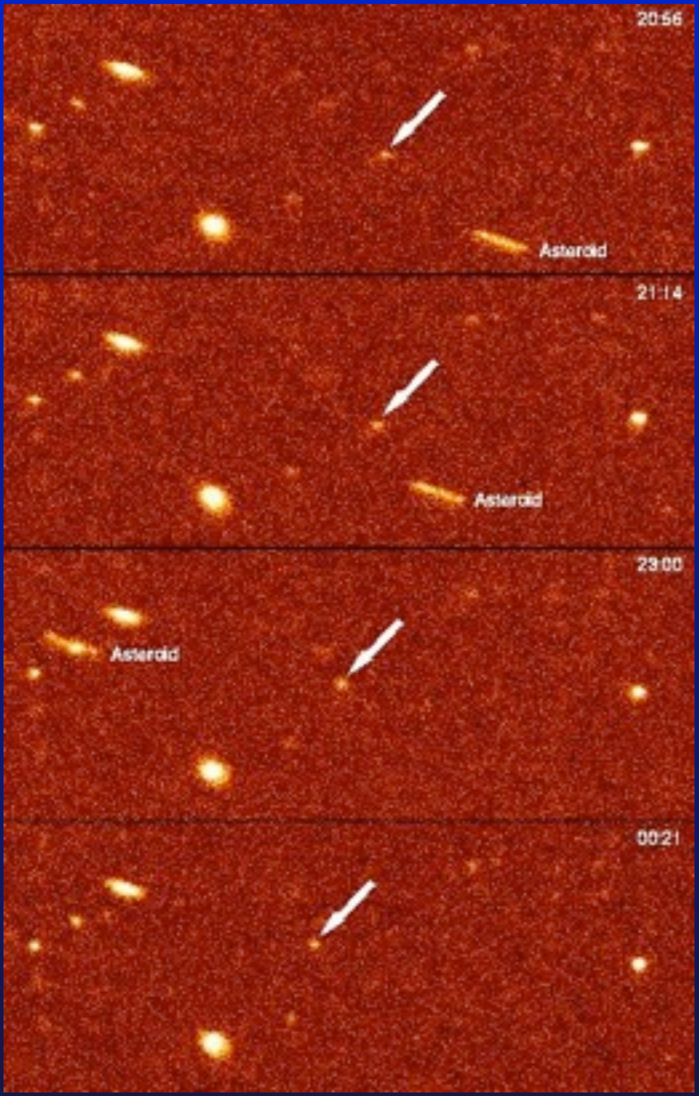


Typical open cluster

- $n_* \sim 4\,\mathrm{stars/pc^3}\ \ (R_{1/2} \sim 2\,\mathrm{pc})$
- $t \sim 200\,\mathrm{Myr}$
- $\langle v_*^2 \rangle^{1/2} \sim 1\,\mathrm{km/s}$

Fernandez & Brunini 00

Discovery of Kuiper Belt Object (KBO) #3

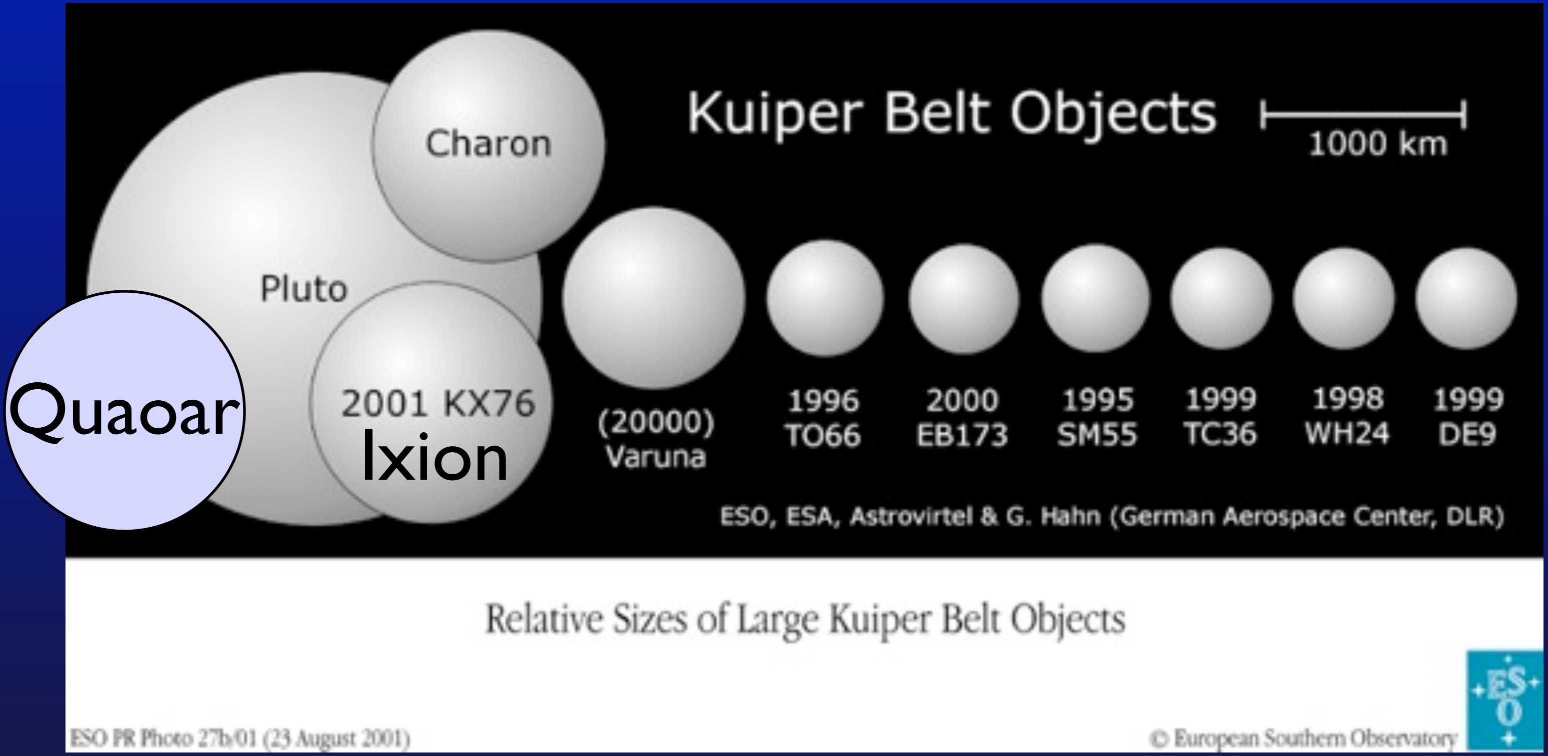


1992 QB1: “Smiley”

Figure 4.1 Jane Luu and Dave Jewitt. The picture was taken in the control room of the UKIRT telescope in 1994. (Jane Luu.)

David Jewitt (U Hawaii)
Jane Luu (UC Berkeley)

Size depends on observed brightness and intrinsic reflectivity (albedo)



Relative Sizes of Large Kuiper Belt Objects

ESO PR Photo 27b/01 (23 August 2001)

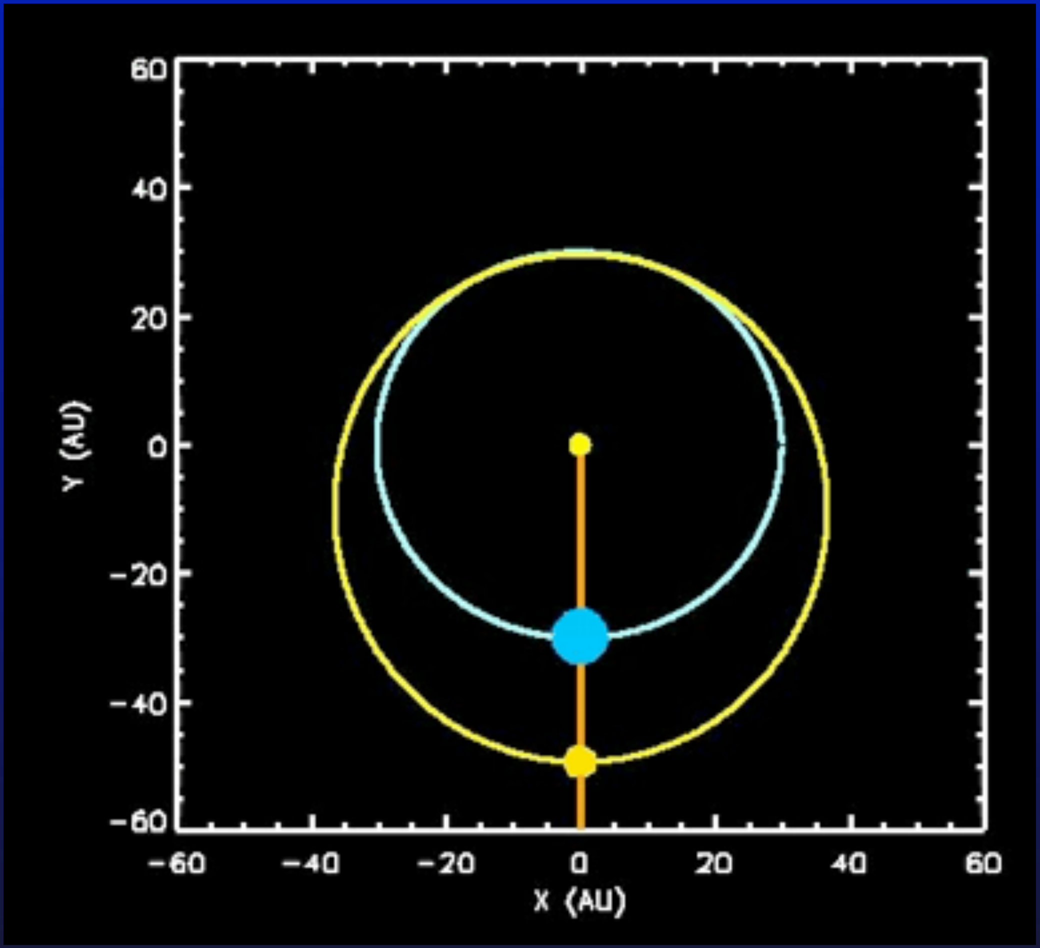
© European Southern Observatory

Planetary Protection Mechanism: Orbit-Orbit Resonance

Neptune makes 3 orbits

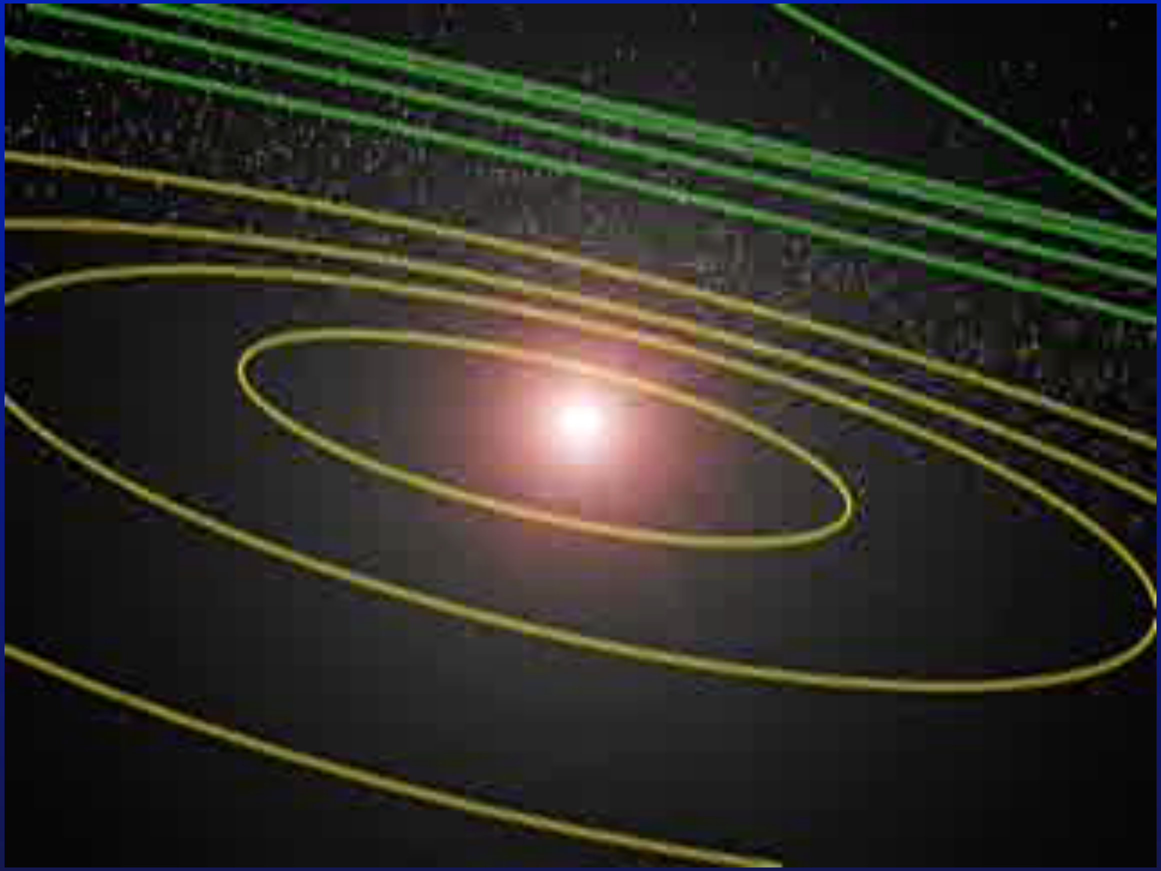
for every

2 orbits of Pluto

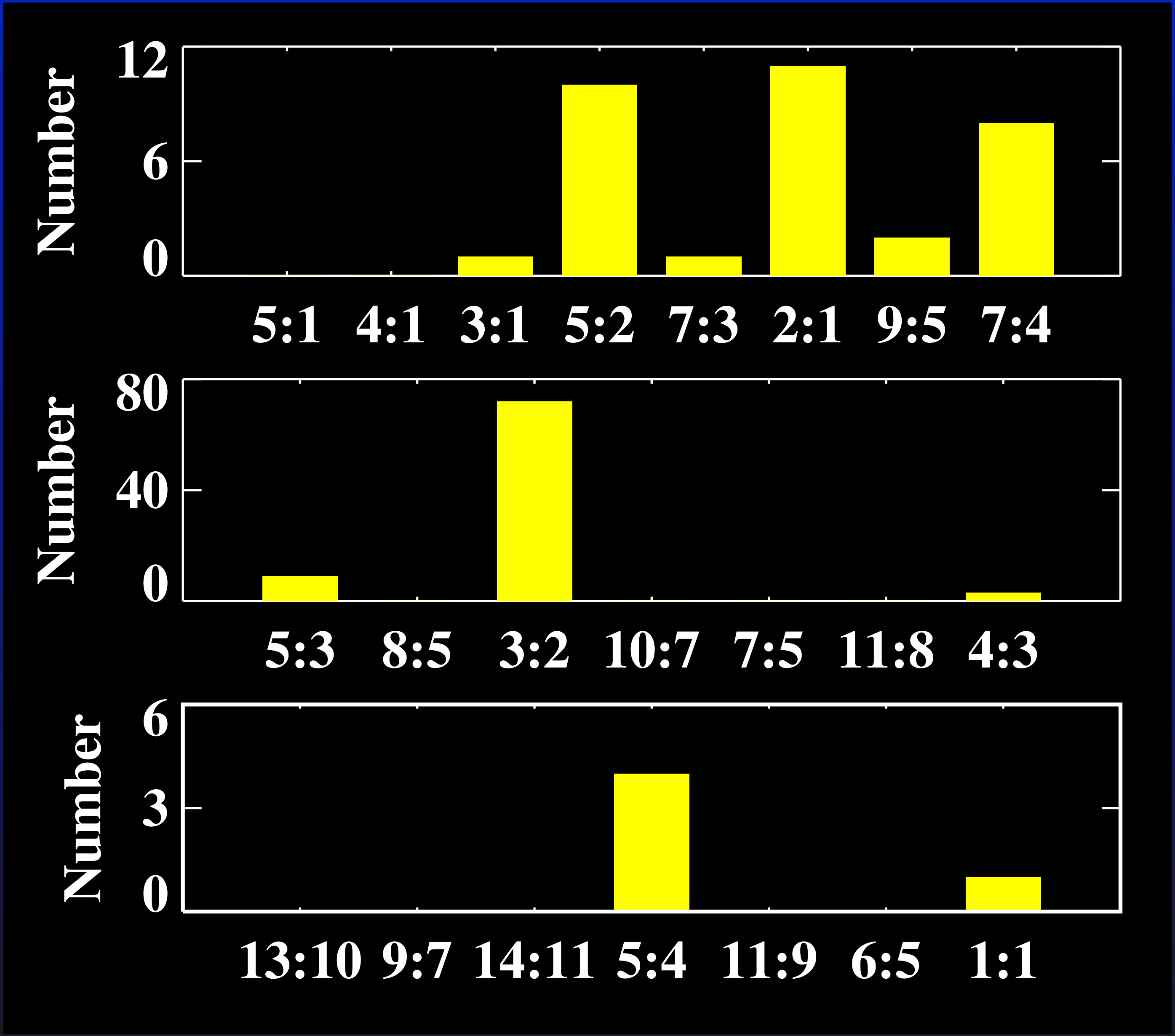


"Dance of the Plutinos"

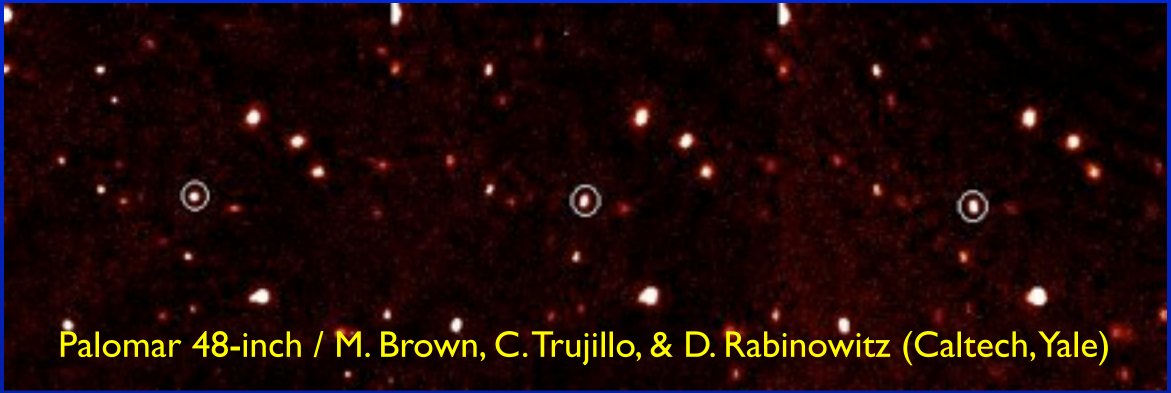
The Orbit of Sedna



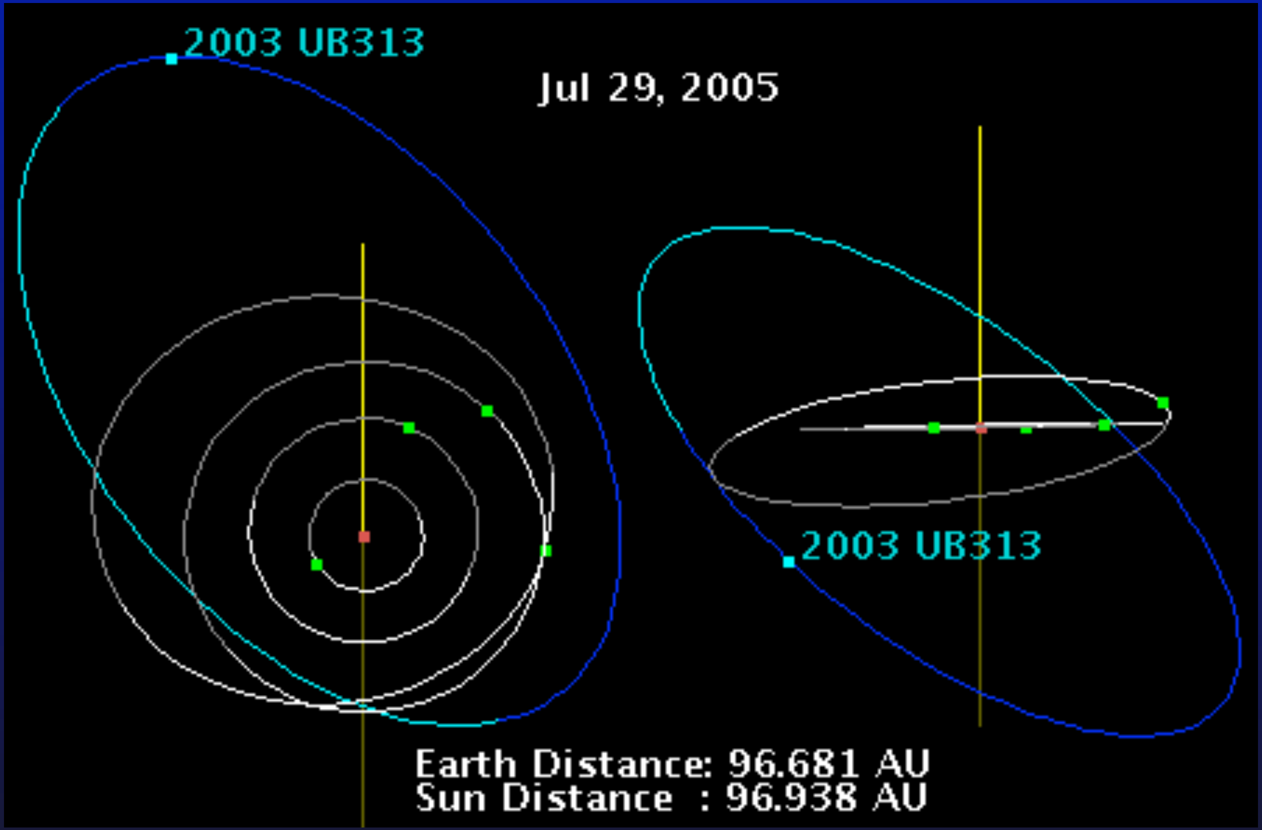
Resonant KBOs

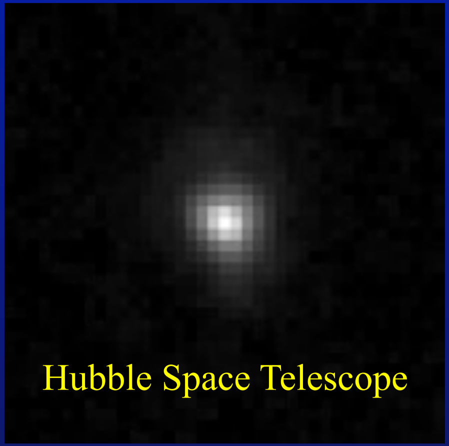


2003 UB313
“Xena”
Bigger than Pluto



Palomar 48-inch / M. Brown, C. Trujillo, & D. Rabinowitz (Caltech, Yale)





Hubble Space Telescope

2003 UB_{313} (m_{app} = 19)
Diameter = 2397±100 km

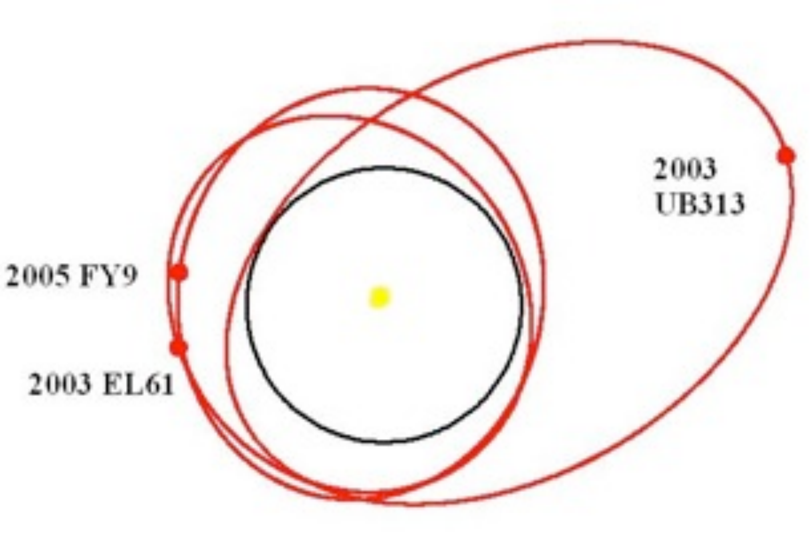
vs.

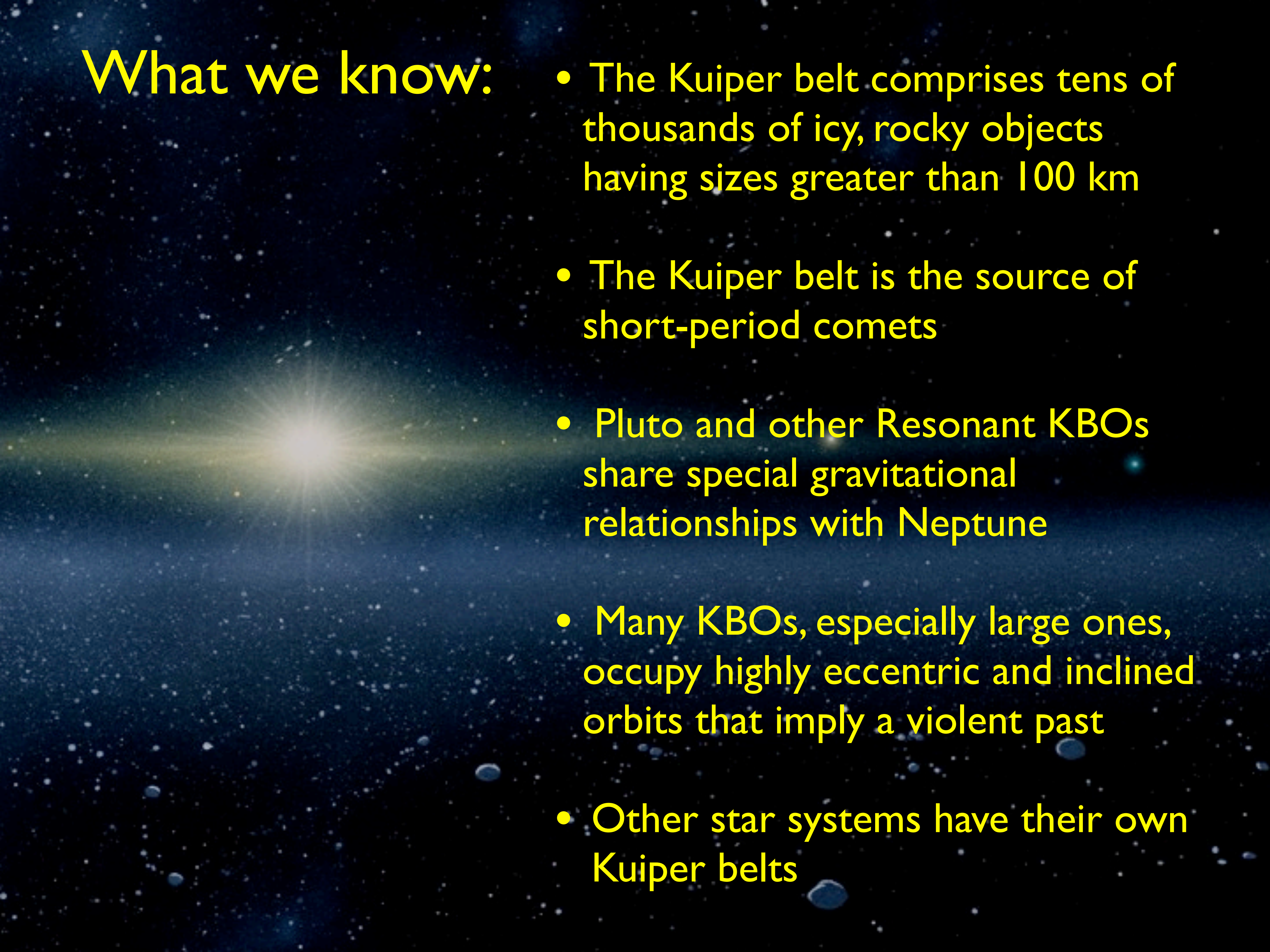
Pluto
Diameter = 2274 km

“Dwarf Planets”

I.A.U. definition

(a) orbits the Sun
(b) hydrostatic (round) shape
(c) not a satellite
(d) not cleared its neighborhood

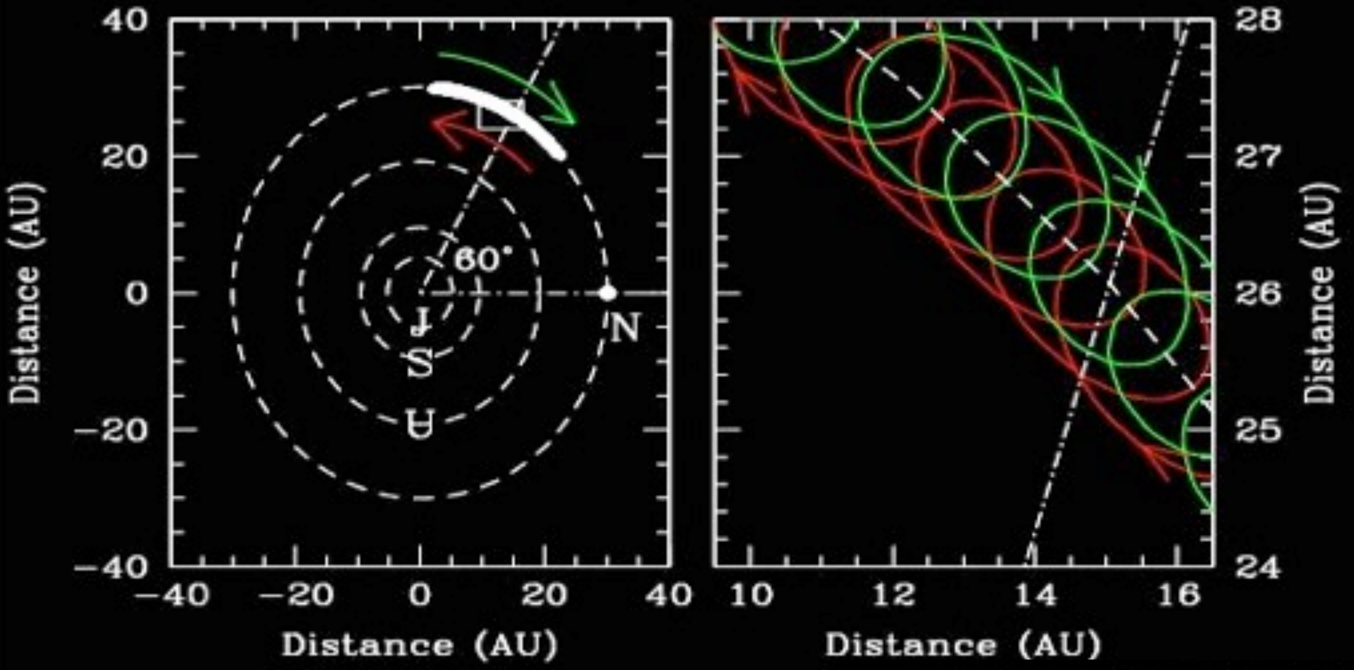




What we know:

- The Kuiper belt comprises tens of thousands of icy, rocky objects having sizes greater than 100 km
- The Kuiper belt is the source of short-period comets
- Pluto and other Resonant KBOs share special gravitational relationships with Neptune
- Many KBOs, especially large ones, occupy highly eccentric and inclined orbits that imply a violent past
- Other star systems have their own Kuiper belts

First discovered Neptune Trojan (1:1)

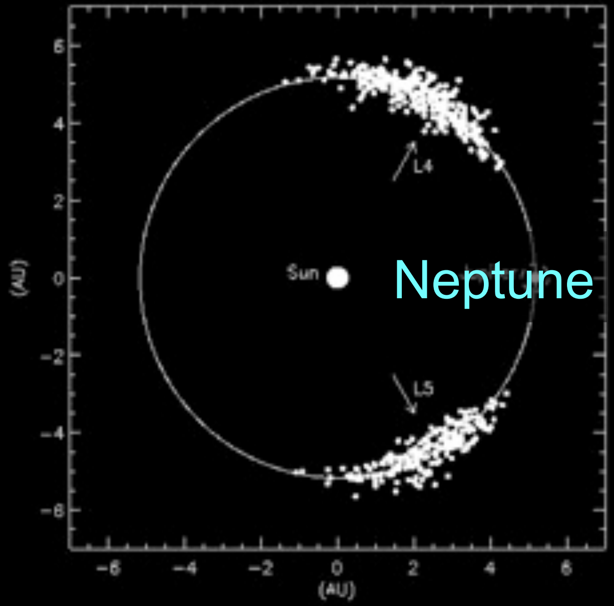


1 “Large” Neptune Trojan in 60 □°

⟹ ~10-30 Large Neptune Trojans

vs. ~1 Large Jovian Trojan

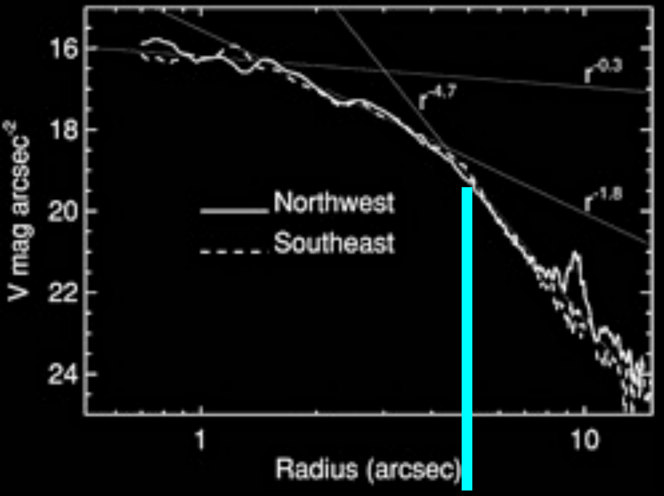
(“Large” ≡ 130-230 km diameter assuming 12-4% visual albedo)



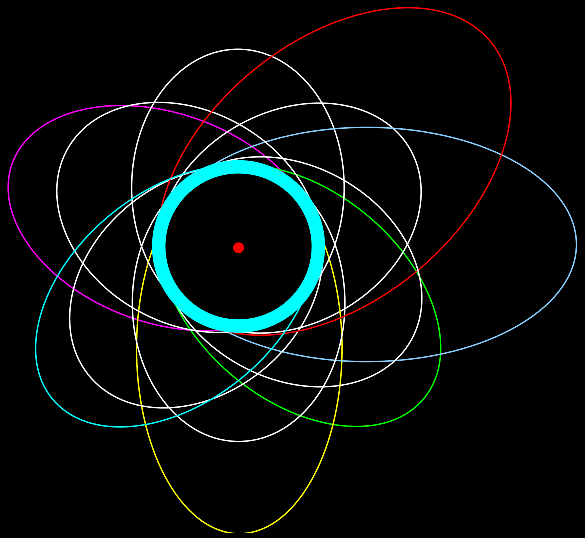
based on Mike Brown's survey limits:
1 Earth mass at less than 200 AU
1 Neptune at less than 500 AU
1 Jupiter (in reflected light) at less than 1000 AU

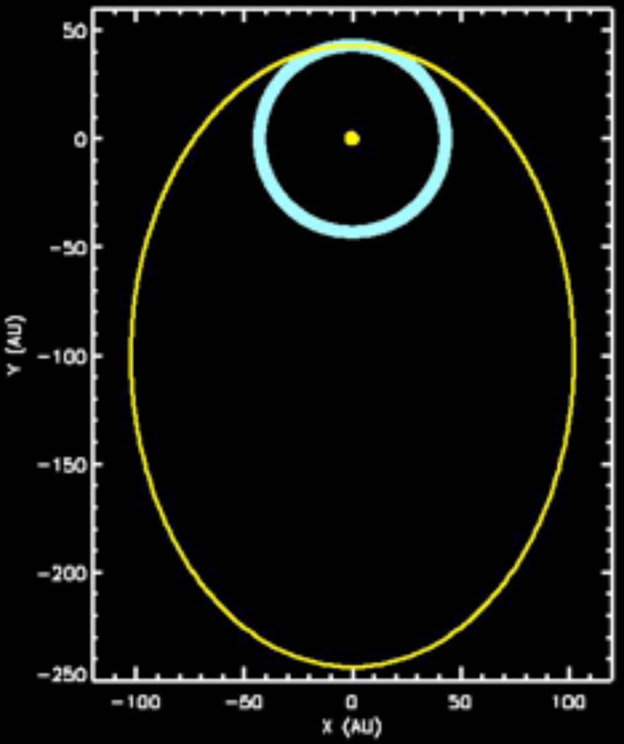
based on Hipparcos and Tycho-2
cannot be a self-luminous main-sequence star above the hydrogen burning limit

infrared detection of a Jupiter or brown dwarf could be interesting

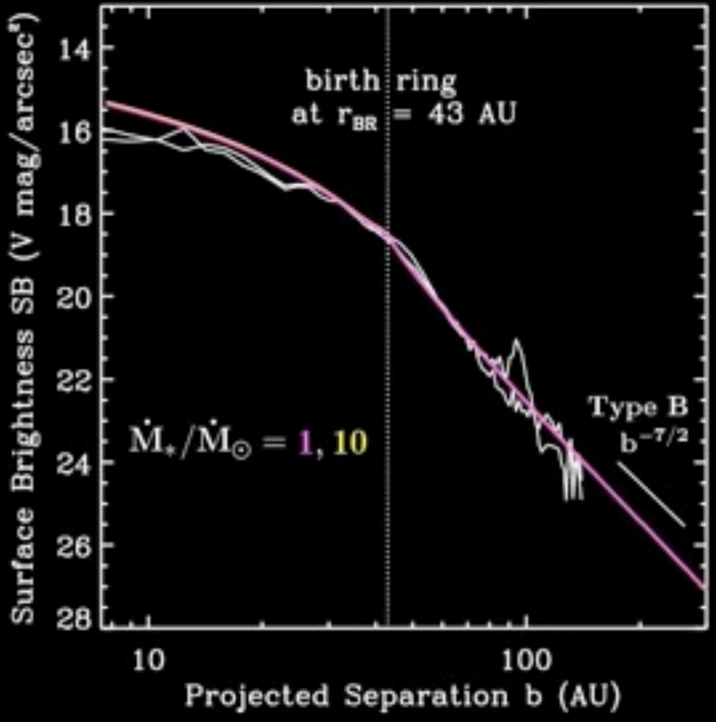


16
18
20
22
24
V mag arcsec^{-2}
1
10
Radius (arcsec)
Northwest
Southeast
$r^{-0.3}$
$r^{-4.7}$
$r^{-1.8}$

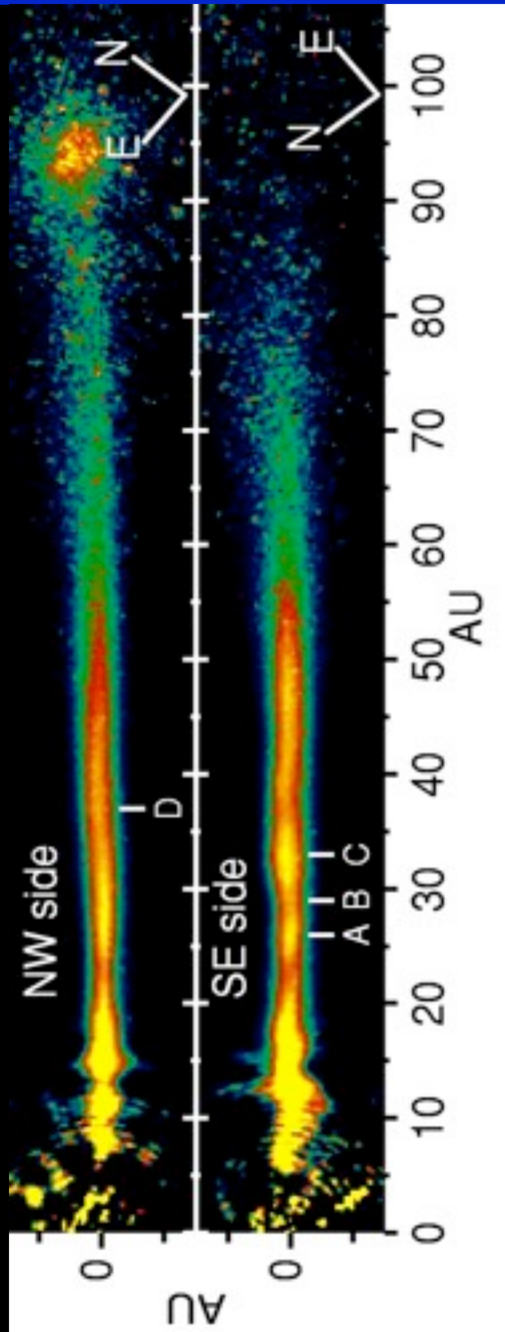




50
0
−50
−100
−150
−200
−250
Y (AU)
−100
−50
0
50
100
X (AU)

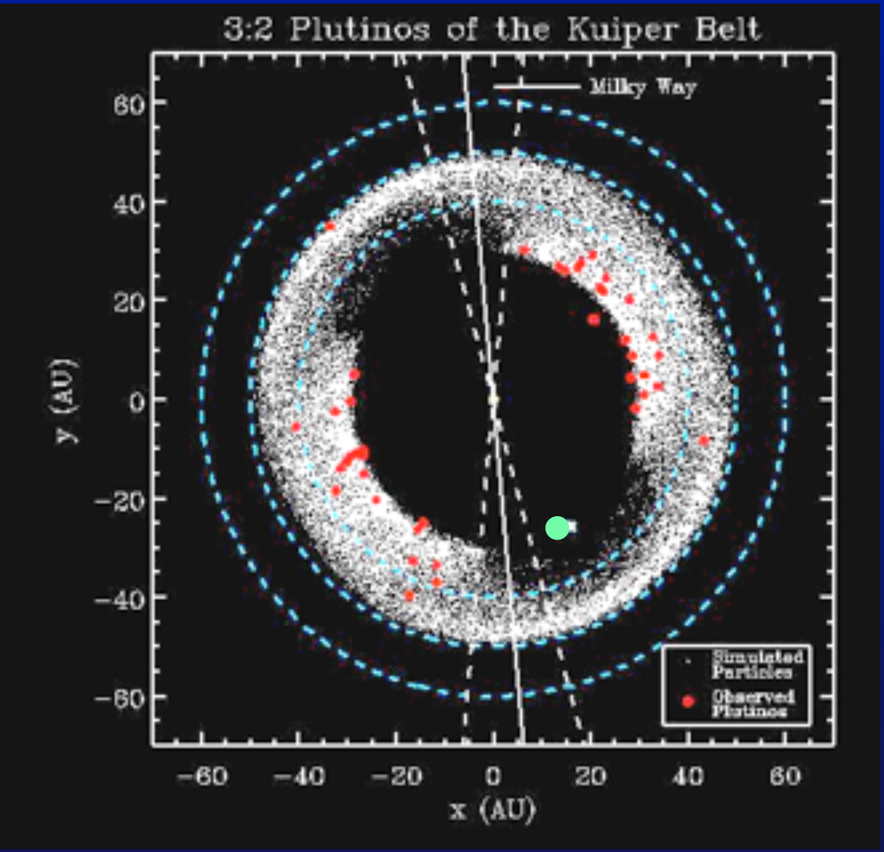


14
16
18
20
22
24
26
28
Surface Brightness SB (V mag/arcsec^2)
10
100
Projected Separation b (AU)
birth ring
at r_{BR} = 43 AU
$\dot{M}_*/\dot{M}_\odot$ = 1, 10
Type B
$b^{-7/2}$



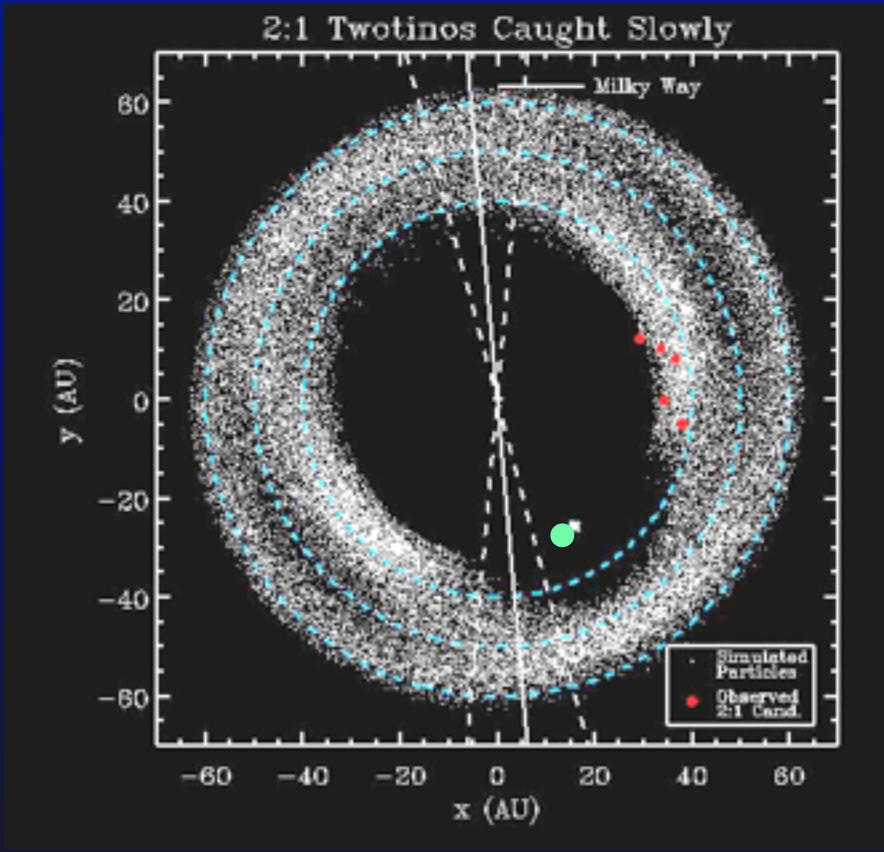
N
E
N
E
NW side
SE side
D
A B C
0
10
20
30
40
50
60
70
80
90
100
AU
0
0
AU

Theoretical Snapshots of Resonant KBOs



⇐ Plutinos 3:2

Twotinos 2:1 ⇒



Chiang & Jordan 2002

Observational Facts and Theoretical Deductions

1. Pluto is the largest known member of a swarm of billions of outer solar system bodies that supply new comets.
2. Pluto and the Plutinos are locked in an orbital resonance established by Neptune.
3. The orbits of many Kuiper Belt Objects are dynamically excited.
4. Pluto is not alone in having an orbital companion.