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On the Definition of Albedo and Application to Irregular Particles

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Summary. The various definitions of albedo used in planetary astronomy are reviewed. In particular, the Bond albedo, which refers only to the reflected and refracted components, is not applicable to small particles or highly irregular particles, where diffraction is not restricted to a well-defined lobe at small scattering angles. Measured scattering functions for irregular particles are presented in a normalized form and are applied to the case of zodiacal light.

Key words: albedo – comets – interplanetary dust – scattering – zodiacal light

1. Introduction

The albedo of an object is a useful parameter in planetary astronomy to characterize the surface or atmospheric properties of the object as well as in the computation of its energy balance. Albedo is also useful in application to the scattering by small particles, such as the zodiacal light or the dust comae of comets, where one wishes to convert from the observed scattered light intensity to the number density of particles. Confusion results, however, when the terminology used in planetary astronomy, particularly the Bond albedo, is applied to dust, where the particle size may be only a few times the wavelength of light and thus the diffraction term cannot generally be defined.

The scattering functions of small particles have generally been computed by Mie theory, strictly valid only for spheres. Exact calculations are also possible for spheroids (Asano and Yamamoto, 1975). Furthermore approximate solutions and semi-empirical approaches of the scattering process are available, among those Oguchi (1973), Purcell and Pennypacker (1973), Giese et al. (1978), Yeh and Mei (1980), Chiappetta (1980), Pollack and Cuzzi (1980), and Greenberg and Gustafson (1981). For randomly oriented ensembles of particles strongly deviating from spherical shape laboratory experiments are still the most powerful tool for scattering problems. Among others such measurements have been performed by Holland and Gagne (1970), Pinnick et al. (1976), Perry et al. (1978), and transformed into the microwave region by Zerull et al. (1974, 1977, 1980), and Greenberg and Gustafson (1981).

Recently loose particle agglomerates have been suggested to explain the observed scattering characteristics of the zodiacal light, either fluffy particles of slightly absorbing material (Giese et al., 1978) or dielectric "bird's nest" structures (Greenberg and Gustafson, 1981). Brownlee (1978) has collected micrometeorites in the stratosphere; many of these indeed have highly irregular shapes similar to the fluffy particles.

The purpose of this paper is to clarify the various definitions of albedo and to present the scattering functions of irregular particles in a normalized form such that they can be directly compared with observations.

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2. Scattering Function and Albedo

A particle of geometric cross section, G, illuminated by a parallel beam of irradiance I_0(erg cm^{-2} s^{-1}) scatters into the solid angle $d\omega = \sin\theta d\theta d\phi$ during the time dt the energy

$$dE = I_0 \sigma(\theta, \phi) d\omega dt, \tag{1}$$

where $\sigma(\theta, \phi)$ is the differential scattering cross section (cm^2 sterad^{-1}), sometimes called the scattering function. It is related to the dimensionless scattering function i defined for spheres (van de Hulst, 1957, p. 35) by $\sigma = (\lambda/2\pi)^2 i$.

The total power scattered in all directions is

$$W_{\text{sca}} = I_0 \int_{4\pi} \sigma(\theta, \phi) d\omega = I_0 C_{\text{sca}}, \tag{2}$$

where $C_{\text{sca}} = Q_{\text{sca}} \cdot G$ is the total scattering cross section of the particle and Q_{sca} the scattering efficiency. In an analogous way one defines $C_{\text{abs}} = G \cdot Q_{\text{abs}}$, the cross section for absorption, and $C_{\text{ext}} = C_{\text{sca}} + C_{\text{abs}}$, the cross section for extinction.

The albedo A of a single particle is defined in the most general way (van de Hulst, 1957, p. 183) as the ratio of the energy scattered in all directions to the total energy removed from the incident beam,

$$A = I_0 C_{\text{sca}} / I_0 C_{\text{ext}} = Q_{\text{sca}} / Q_{\text{ext}}. \tag{3}$$

This definition includes all components (diffracted, refracted, reflected) of the scattered radiation and is also applicable for small particles whose size is comparable with the wavelength.

Another definition of albedo, the Bond albedo, is meaningful in all cases, where the diffraction term σ_d can be separated from the term σ_r due to reflection and refraction. Therefore diffraction must be restricted to an extremely narrow lobe near $\theta = 0°$. This is the case for large smooth particles [when for a particle of radius a the condition $(2\pi a/\lambda)|m-1| > 10$ is satisfied, $\int \sigma_d d\omega \to G$ and $C_{\text{ext}} \to 2$ G, Hodkinson (1966)].

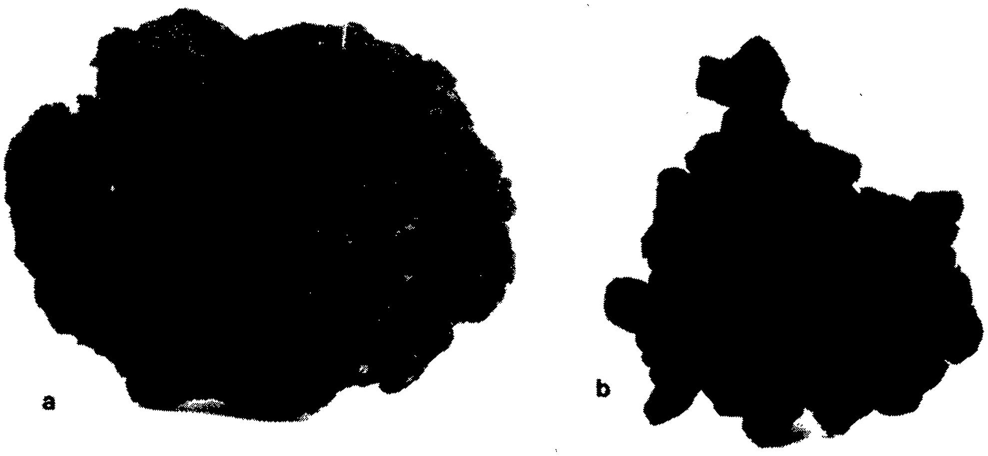


Fig. 1a and b. Typical particle shapes considered: **a** compact particle, **b** fluffy particle

In the case of rough or highly irregular particles the diffraction domain is extended to larger scattering angles [see Chiappetta (1980) and discussion of irregular particles in Sect. 3]. When the restrictions mentioned above are satisfied, however, the Bond albedo A_B, without diffraction, can be defined as the ratio of the energy reflected and refracted by the particle in all directions to the energy incident on the geometric cross section.

$$A_B=\frac{1}{G}\int_{4\pi}\sigma_r(\theta,\phi)d\omega. \tag{4}$$

The Bond albedo is commonly used in planetary astronomy (Moroz, 1968). It is also referred to as the spherical albedo or the whiteness.

One can combine Eqs. (2)–(4) for large particles to obtain the relation

$$A=\frac{1}{2}(1+A_B). \tag{5}$$

Therefore, a large particle has $A=1$ if it is completely white (conservative scattering, $A_B=1$) and $A=0.5$ if it is absolutely black ($A_B=0$). An illustrative example of the dependence of A on the particle size from very small (size $\ll\lambda$; $A\rightarrow 0$) to larger sizes is presented for spheres by Kerker 1969, p. 124.

Another widely used form of albedo is the *geometric albedo* A_P (often denoted as p). It is defined as the ratio of energy scattered by the particle (or planet) at $\theta=180°$ (backward scattering; full phase) to that scattered by a white disk of the same geometric cross section, G, scattering according to Lamberts law ($\sigma_r(180°)=G/\pi$):

$$A_P=\frac{I_0\sigma(180°)}{I_0G/\pi}=\frac{\pi}{G}\sigma(180°). \tag{6}$$

For large particles, where the diffraction component can be separated, the Bond albedo can be directly related to A_P. For symmetry with respect to the direction of incident radiation, $\sigma_r(\theta,\phi)=\sigma_r(\theta)$, we let $\sigma_r(\theta)=\sigma_r(180°)\cdot j(\theta)$. Eq. (4) then becomes

$$A_B=\frac{\sigma_r(180°)}{G}\cdot 2\pi\int_0^\pi j(\theta)\sin\theta d\theta \tag{7}$$

or

$$A_B=A_P\cdot q, \tag{8}$$

where $q=2\int_0^\pi j(\theta)\sin\theta d\theta$ is known as the phase integral in planetary photometry. The phase function $j(\theta)=\sigma_r(\theta)/\sigma_r(180°)$ is often expressed in terms of the phase angle $(\pi-\theta)$ instead of θ.

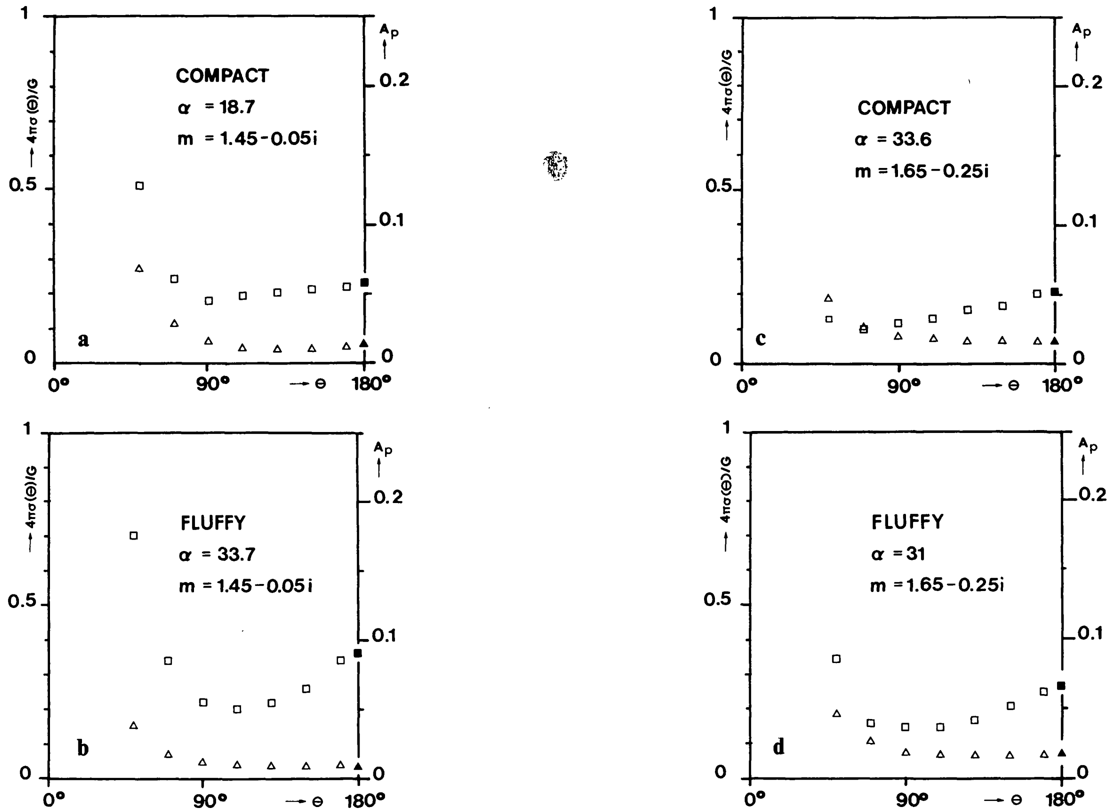
A conservative isotropic scatterer ($A_B=1$) would have a geometric albedo

$$A_P=\frac{G}{4\pi}\cdot\frac{\pi}{G}=1/4. \tag{9}$$

The Bond Albedo, phase integral, and geometric albedo are useful parameters in asteroid and planetary studies. For application to the scattering by interplanetary dust, as discussed in Sect. 4, the Bond albedo is not appropriate since the dust particles do not fulfil the condition size $\gg\lambda$. In many cases, such as cometary observations, the scattering at $\theta=180°$ is not measured directly, nor do the measurements exist over the entire range of scattering angles, so that Q_{sca} and A cannot be directly determined. Instead, one is interested in relating the observed intensity over a limited range in scattering angle to the total number of scattering particles. For this purpose, and in order to discuss the scattering characteristics of irregular particles measured in the laboratory, it is convenient to express the scattering at any angle relative to that of a conservative isotropic scatterer, $\sigma(\theta)/(G/4\pi)$. One may also define an "albedo" at any angle in terms of the backscattering by a white Lambert disc of cross section G, analogous to the definition of geometric albedo, $(\pi/G)\sigma(\theta)$, hereafter denoted $A_p(\theta)$. The scattering by irregular particles, discussed in Sect. 3, will be displayed with these normalizations.

3. Albedo of Irregular Particles

In this section, the albedos of irregular particles derived from microwave analog measurements (Zerull et al., 1974, 1977, 1980) are compared to Mie calculations of spheres with the same mean geometrical cross section. During the scattering measurement, the particle is continuously rotated about its vertical axis. The measurements are repeated for at least 3 orientations of the particle about its horizontal axis. Thus, the averaged final data approximate a monodispersed mixture of randomly oriented particles of specific size and form.



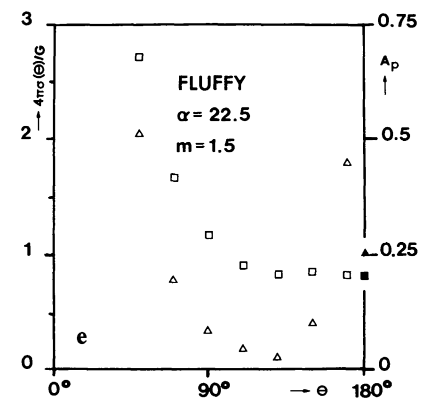


Fig. 2a–e. Normalized scattering functions for irregular particles.
Lower scale: scattering angle θ.
Left scale: scattering cross section normalized to isotropic scatterer [i.e. $4\pi\sigma(\theta)/G$].
Right scale: scattering normalized to the scattering by a white disk of the same geometric cross section [i.e. $A_p(\theta)=\pi\sigma(\theta)/G$]. □: measured scattering function for irregular particle (average over interval of 20°); △: scattering by sphere of equivalent geometric cross section (Mie theory, average over interval of 20°); **a** Compact particle, $\alpha=18.7$, $m=1.45-0.05i$; **b** Fluffy particle, $\alpha=33.7$, $m=1.45-0.05i$; **c** Compact particle, $\alpha=33.6$, $m=1.65-0.25i$; **d** Fluffy particle, $\alpha=31.0$, $m=1.65-0.25i$; **e** Fluffy dielectric particle, $\alpha=22.5$, $m=1.5$. Size parameter $\alpha=2\pi a/\lambda$ refers to sphere of same geometric cross section

Particles deviating in two different ways from spherical structure are considered (Fig. 1). One type is highly irregular, but includes no empty space within their structure; this type is called "compact" (a). The other type resembles particles which are frequently found among the particles of extraterrestrial origin collected by Brownlee (1978). These include considerable spaces within their structure (between 30% and 50% in our examples). They are called "fluffy" particles (b). The averaged density, however, is still higher than 1 g/cm^3, assuming a bulk density typical of rocky material. Two degrees of absorption are chosen, characterized by the refractive indices $m_1=1.45-0.05\,i$ and $m_2=1.65-0.25\,i$. For comparison, the case of a fluffy particle of purely dielectric material ($m=1.5$) was also investigated.

The data are displayed in Fig. 2. The scale on the left gives the scattering normalized to a conservative isotropic scatterer ($G/4\pi$ sterad^{-1}), while the scale on the right gives $A_p(\theta)=(\pi/G)\sigma(\theta)$. Squares refer to the measured irregular particle, triangles to a sphere of equivalent cross section (Mie theory). To smooth out the maxima and minima of the scattering diagramm, which are especially vivid for non- or slightly absorbing spheres, we have averaged the scattering function over intervals of 20°. The laboratory data extend only to 170°. Points at 180° were obtained by linear extrapolation from the original scattering measurements, $\theta=120°$–$170°$ (Weiss, 1977).

The data presented in Fig. 2a–d lead to the following general conclusions, which are substantiated by other laboratory data: the scattering to be expected from irregularly shaped absorbing particles at scattering angles $\geqq 40°$ is markedly higher than that of equivalent spheres. The enhancement observed is stronger in the case of the less absorptive material and is stronger for fluffy particles than for compact ones. The enhancement factor is approximately 5 for the slightly absorbing fluffy particles at θ

Table 1. Comparison of derived zodiacal dust albedo with fluffy particle

θ	S	A_θ, zodiacal light	A_θ, fluffy (2b)
90°	8.0 10^{-23}	0.10	0.055
120°	8.86 10^{-23}	0.11	0.052
150°	11.6 10^{-23}	0.15	0.065
180°	18.4 10^{-23}	0.23	0.091

S: volume scattering function for $n(r) \propto r^{-1.3}$ (cm^2 sterad^{-1} cm^{-3})
$A_\theta = \frac{\pi}{G}\,\sigma(\theta)$

>90°. In all cases, 2a–d, an increase toward backscattering is seen for the irregular particles. This feature can be explained by shadowing and multiple reflection effects, as described in detail elsewhere (Wolff, 1975; Giese et al., 1978; Mukai, 1980). The general increase of intensity at medium and large scattering angles may be explained as follows:

1. Whereas absorbing spheres absorb nearly all refracted radiation, this radiation has a much better chance to reach the observer in the case of irregular, especially fluffy, structures (see Giese et al., 1978; Zerull et al., 1980).

2. Whereas the forward lobe of a sphere can be satisfactorily approximated by calculations of the diffraction of a spherical disk, the conditions are much more complicated in the case of highly irregular structures.

A fissured cross section contains many smaller diffraction centers which preserve their typical features at least partly, i.e., diffraction is not restricted to a well-limited domain of small scattering angles. Furthermore, the problem can no longer be treated as two-dimensional; for instance, diffracted rays caused by outstanding parts of an irregular particle can be scattered by parts behind. All these effects extend the diffraction domain and diffraction becomes non-negligible up to the backscattering region. This complex interaction may cause a deviation from $Q_{ext} \to 2$ for large, highly irregular particles, as observed in extinction measurements by Wang (1980). In most cases, the observed Q_{ext} increased, consistent with the enhanced scattering. Consequently, the particles do not meet the definitions of the Bond albedo, because the diffraction cannot be unambiguously separated, even for large particles.

The scattering by a purely dielectric fluffy particle is shown in Fig. 2e. In this case, there are no true shadowing effects, because no absorption takes place within the dielectric particle. Consequently, no increased backscattering can be observed for the fluffy particle, although there is still a general enhancement of intensity at medium scattering angles.

The microwave scattering data presented here correspond to particle diameters 1–5 µm at optical wavelength ~0.5 µm. Scattering measurements currently being carried out by a means of a laser for much larger particles (~50 µm diameter) substantiate the general enhancement at medium and large scattering angles.

For particles too small to fulfil the condition $\alpha|m-1| > 10$, Q_{ext} for an irregular particle can differ from that of an equivalent sphere. In particular, the first maximum in Q_{ext} for spheres at $2\alpha|m-1| \approx 4$ is not present for irregular particles (Hodkinson, 1966). Greenberg (1972) has discussed the effects of particle shape on the absorption and emission cross sections for small particles. He showed that important shape effects may exist for elongated particles even in the Rayleigh limit. Microwave scattering measurements on compact dielectric particles of various shapes (but approximately equidimensional) have shown that the scattering at $\theta > 40°$ starts to deviate substantially from spheres when $\alpha \approx 5$ (Zerull et al., 1974, 1980.).

4. Discussion

The scattering functions of irregular particles have several potential applications in solar system astronomy, including zodiacal light, comets, and planetary rings. Giese et al. (1978) have suggested that fluffy, slightly absorbing particles may be a good representation of interplanetary dust particles, based on the similarity of their scattering function and polarization to that of the zodiacal light. In the case of zodiacal light, one observes the scattering integrated along the line of sight. These observations can be inverted to yield the volume scattering function of the dust, equal to the number density of particles times the average differential scattering cross section per particle (Leinert et al., 1976; Dumont and Sanchez, 1975).

The size distribution and spatial density of the interplanetary dust derived from lunar microcrater statistics and in situ measurements have been discussed by Morrison and Zinner (1977), Fechtig et al. (1974), Fechtig (1976), and LeSergeant and Lamy (1980). Giese and Grün (1976) showed, based on the Fechtig size distribution, that the main contribution to zodiacal light comes from particles 10 µm–100 µm radius, with grains <1 µm radius contributing <1%. Hanner (1980) demonstrated that Giese and Grün's "maximum" model leads to a geometric albedo of 0.24, higher than suggested by the dark appearance of collected micrometeorites and the infrared brightness of the zodiacal light.

We can make a similar comparison between the derived albedo of the zodiacal dust and that of the irregular particles. Table 1 compares $A_p(\theta) = (\pi/G)\,\sigma(\theta)$ for the slightly absorbing fluffy particle (Fig. 2b) with $A_p(\theta)$ based on the volume scattering function of the zodiacal light derived for $n(r) \propto r^{-1.3}$ and the Giese-Grün "maximum" size distribution from the Fechtig lunar microcrater curve. The zodiacal light values are about a factor of two higher than the fluffy particles, supporting Hanner's conclusion that the Fechtig flux curve is too low. Clearly, if the scattering function for spheres were assumed, the discrepancy would be larger.

One of the possible explanations for this difference is a higher contribution from small particles. Indeed, the lunar microcrater size distribution of Morrison and Zinner, re-examined by LeSergeant and Lamy (1980), has a large population of submicron particles, compared to that given by Fechtig. The potential contribution of this population of small grains to the zodiacal light has been discussed by Lamy and Perrin (1980). If they assume Fresnel reflection with an albedo of 0.05 for the large grains and Mie scattering by iron spheres for the submicron grains, they find that the submicron grains dominate the optical scattering. If, however, they introduce the enhancement due to fluffy particles for the larger grains, the two components become comparable. As Lamy and Perrin point out, the neutral color of the zodiacal light indicates that submicron grains cannot be dominant. If further research substantiates this high population of submicron grains, then scattering by fluffy, slightly absorbing particles offers a reasonable explanation why these small grains do not dominate the optical scattering. In any event, the enhancement at medium scattering angles in the scattering by irregular particles seems necessary to reconcile the zodiacal light brightness with the in-situ flux measurements.

Fluffy particles may also be a reasonable model for cometary dust. The size distribution of dust released from comets derived by Sekanina from analysis of dust tail dynamics (Sekanina and Miller, 1973; Sekanina, 1980) gives a mean size parameter similar to that of the irregular particles displayed in Fig. 2. Consequently, the normalized scattering functions in Fig. 2 are directly applicable to the interpretation of cometary observations. Giese (1980) showed that the scattering function (and polarization) measured for Comet West (Ney and Merrill, 1976) are compatible with mixtures of micron-size irregular dielectric and absorbing grains and that even inhomogeneous fluffy particles containing both, dielectric and absorbing constituents in one agglomerate, are also likely candidates to explain the observed scattering- and polarization behaviour.

Strong scattering efficiencies of Saturn ring system regions at smaller scattering angles, as observed by Voyager I (Smith et al., 1981), also indicate particles sizes as discusses in this paper. Therefore, the scattering results of irregular particles may also be relevant for the interpretation of these data.

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