Interaction of Interplanetary Dust Grains with the Solar Radiation Field

Ph. L. Lamy

Cornell University, Ithaca, N. Y., Laboratoire d'Astronomie Spatiale, Marseille

Summary. The interaction of interplanetary dust grains with the solar radiation field is investigated for micronic and submicronic spheres of quartz, obsidian, andesite, ice and iron. The dependence of the complex indices of refraction on wavelength was taken into account using Mie theory to obtain the efficiency factors for absorption and radiation pressure. The temperature distribution differ markedly from past results, and show that silicate grains can come very close to the Sun, thus justifying the existence of a dusty component of the solar corona up to approximately $2R_\odot$.

Improved vapor pressure formulae yield increased lifetimes for interplanetary grains. It is shown that the onset of rapid destruction by sublimation is extremely sudden and has a well-defined location which depends upon the absorption character of the grains' material. Finally, the radiation pressure force never exceeds the gravitational force for silicates and ice—a result which contradicts general belief; it does so for iron, but only for a narrow range of values of the grain's radius. These results call for a new investigation of the dynamics of interplanetary dust particles.

I. Introduction

This article is part of a continuing effort to improve our understanding of the behavior of micronic and submicronic interplanetary dust grains. It is devoted to the study of the interaction between these grains and the solar radiation field, and thus includes three distinct problems:

i) the determination of the temperature distribution $T(R, s)$ of a grain of given material as function of the heliocentric distance R and its radius s,

ii) the rate of erosion as controlled by sublimation,

iii) the radiation pressure force acting on the grains.

The second problem is related to the first in the sense that the sublimation rate depends upon the temperature; however, it requires an investigation of its own. The third problem includes a very familiar question, that of deciding whether or not the radiation pressure force F_p exceeds the solar gravitational attraction F_g (according to a frequent misconception, this causes the dust to be blown out of the solar system).

Similar work in the past has always encountered difficulty in assigning correct cross-sections for absorption and radiation pressure to the grains since their sizes are of the same order as the interacting wavelengths. It is well known that the exact solution, restricted to spherical particles, was given by Mie and completed by Debye for the problem of radiation pressure (see Van de Hulst, 1957, for an authoritative discussion). The intricacy of Mie theory has led most of the researchers to work out approximate solutions thus preventing any meaningful advance on the problem of interplanetary grains. The lack of data pertaining to the optical properties of those materials which would be possible candidates for interplanetary grains is certainly partly responsible for the past situation.

Recent improvements in this field together with new Mie scattering computations—see Irvine and Pollack (1968), Pollack et al. (1973) and Lamy (1974a)—allow one to consider very realistic candidates for interplanetary dust and to solve for their interaction with the solar radiation field in a more accurate manner.

The reference work on temperature distribution on interplanetary grains is that of Over (1958) who considered quartz particles. His idealization of a uniform absorption spectrum in the infrared (beyond $3\,\mu$) allowed him to avoid using Mie theory but leads to large errors as will be seen later.

Swamy and Donn (1968) used Mie theory to obtain the temperature of submicronic grains of "dirty-ice", iron, "impure quartz" and graphite between 0.1–1 a.u. The curious temperature reached by their ice ($T \geq 450\,°K$) raises some questions about their calculation procedures, especially since the indices of refraction used seem rather unreliable or grossly approximated, and no details are given on the computational methods such as the spectral range of integration. Kaiser (1970) in his thorough study of the F-corona, classified spherical grains of quartz as small, Rayleigh, and large, to approximate their cross-section for

absorption and derive their temperature up to a heliocentric distance of 100 solar radii.

The literature on the radiation pressure problem is more abundant: Elsässer and Schmidt (1966), Aller *et al.* (1967), Singer and Bandermann (1967) touched upon it but their work is overshadowed by three major contributions which require further comments. Shapiro *et al.* (1966) obtained the ratio $\beta = F_p/F_g$ as a function of the grains' radius s for quartz, water and iron spheres, unfortunately without giving much detail on their data and solution. A critical evaluation of their results is therefore difficult but their good agreement with those presented here make them very interesting. Greenberg (1967) used Mie theory with wavelength dependent index of refraction to compute the radiation pressure on "dirty-ice" grains as a function of the temperature of a blackbody radiation field. This is not very practical for the Sun since the blackbody approximation yields large errors as will be explained later. Gindilis *et al.* (1969) assumed a constant index of refraction for quartz, titanite and iron but took the dependence on wavelength into account for water and graphite. Their thorough investigation led to very useful results and considerable insight into the problem.

In this study, we consider micronic and submicronic spheres of quartz, obsidian, andesite, water-ice and iron and solve for their interaction with the solar radiation field on the basis of Mie theory with the wavelength dependence of the indices of refraction taken into account. Part II presents the scope of the problem, the general assumptions and their limitations. Part III covers the problem of temperature distribution for the various models of grains and prepares for the investigation of their destruction under sublimation, and the determination of their lifetimes, in Part IV. Finally Part V presents the results for the radiation pressure force.

II. Assumption and Techniques of Solution

All particles that are considered in this report are idealized models which are, of course, not likely to be completely similar to interplanetary dust particles. For instance, real grains are probably not made of one pure substance but of a mixture of several, have arbitrary shapes and surfaces affected by collisions, sputtering, radiation damage and local impurities. Hence, before going further, we wish to mention the assumptions used to idealize the problem and to carry out solutions; these are presented below, together with their limitations and a discussion of the validity of the results to which they lead.

First, the particles are assumed spherical for simplicity and because solutions to Maxwell's equations for an arbitrarily shaped body in an electromagnetic field are not available. However, it is a well known fact that potato-shaped (i.e., irregular but roughly spherical)

bodies scatter like spheres. Therefore, if, as believed, interplanetary particles are of such shape since collisions and sputtering should cause sharp surface protuberances to be removed, this restriction will not limit the validity of the results much. If, however, skeleton shaped particles do exist, Mie scattering will not give acceptable approximations as such bodies are likely to scatter light diffusively. Greenberg and his co-workers have devoted much effort to the study of the effect of surface roughness on scattering, on both theoretical (Greenberg, 1968) and experimental (Greenberg *et al.*, 1971) grounds. Their results tend to support the validity of the statement on potato-shaped grains although important discrepancies may occur in some special cases. However, a further study by Greenberg and Shah (1971) on shape considerations in the determination of the temperature of interstellar particles showed a negligible influence. In an extreme case (infinite cylinders, thin needles, flat disks), the discrepancy in the temperature was only 10% compared to spheres.

Studies of this kind always include the consideration of materials of dielectric and metallic types. In this respect, quartz and iron are the archetypes and are favored because of their predicted abundance among elements and because several authors—cf. Wolstencroft and Rose (1967)—explain the polarization of the Zodiacal Light by a mixture of iron and quartz particles. However, one may raise doubts about the existence of pure crystalline quartz in interplanetary space. Obsidian, a volcanic glass, represents a good example of dirty fused quartz and was selected aolng with andesite, a rock close to the olivine family representative of stony meteorites to offer more realistic models of dielectric type interplanetary dust. Water-ice is specifically studied here because it is supposed to be injected in interplanetary space by comets according to Whipple's models and also because if "particles" of water are present, they will be in the solid phase rather than the liquid for, as will be seen later, all ice particles are sublimated before they reach the melting point. These five materials, whose physical properties are summarized in Table 1, present the decisive advantage of having their complex indices of refraction reasonably well known in the visible and infrared.

Table 1. Physical properties of the five materials. (The densities of obsidian and andesite are averages only; their melting and boiling points were not found in the literature reviewed. Taking them to be of the same order of magnitude as quartz should be a sufficient approximation)

	Quartz	Obsidian	Andesite	H_2O-ice	Iron
Molecular or atomic mass	60	67	69.7	18	56
Density (g/cm³)	2.65	2.37	2.47	0.92	7.86
Melting point (°K)	1983			273.2	1808
Boiling point (°K)	3070			373.2	3023

Therefore it has been possible to apply Mie theory to solve for the interaction with an electromagnetic field (Lamy, 1974a) taking into account the precise wavelength dependence of the complex index of refraction. Inherent to the theory are the well-known assumptions of optical smoothness of the grain's surface and its overall electrical neutrality—a condition clearly violated since interplanetary particles have a potential of about 5—10 volts (Belton, 1966).

Interplanetary grains experience large temperature variation from the outer part of the solar system to the corona. It is known that optical properties of any material vary with its temperature; this problem was thoroughly investigated by Lamy (1974a) who concluded that the actual lack of data prevents taking this variation into account but that its is fairly negligible between— $100° C$ and $400° C$ approximately for all materials except ice, for which the range is $-80° C$ to $0° C$. Results have nevertheless been derived for temperatures outside these ranges and should be considered to give approximative behavior only.

The Sun is obviously of utmost importance in the present study and it is necessary to specify its emission spectrum $F_\odot(\lambda)$. The source of energy will be the conventional solar spectrum while the energy flux from corpuscular radiation is neglected. Although the 6000 °K black-body model is realistic and very convenient as far as the numerical integration is concerned, it is inaccurate by as much as one order of magnitude in the region $0.2 < \lambda \leq 0.6 \mu$, a fact of considerable importance for the radiation pressure. Therefore, in this region, $F_\odot(\lambda)$ was obtained from the specific values given by Nicolet (1951). For $\lambda > 0.6 \mu$, the black-body approximation is perfectly acceptable as can be seen from the curve of the solar energy spectrum given by Malitson (1965) in her review. Discrepancies are minute and lead to only slighty overestimated values.

III. Temperature Distribution of Interplanetary Dust Grains

a) Equation for the Temperature

The local thermal equilibrium of a particle implies that the solar energy absorbed is balanced by the energy radiated outward by the particle and the energy removed by sublimation. The ranges of temperatures considered for the various substance will be below the melting point; therefore, only sublimation is expected to occur, if it is significant at all. The following reasonable assumptions are made in order to get an equation simple enough: the particle will have a definite temperature which is the temperature of quasi-equilibrium. This is perfectly acceptable since the thermal skin depth will equal the size of the grains in a very short period of time so that the heating is isotropic when viewed over the time scales of interest here. The heat flow by conduction inside the grain is neglected

and no change of phase is supposed to occur. The energy absorbed by a grain of radius s during one second is

$$\left(\frac{\Omega}{\pi}\right) \pi s^2 \int_0^\infty Q_{abs} F_\odot(\lambda) \, d\lambda$$

where Q_{abs} is the efficiency factor for absorption, $F_\odot(\lambda)$ is the monochromatic emissive power (or flux) of the Sun, and Ω is the solid angle subtended by the Sun at the point where the particle is located. If R is the distance from the Sun and $R_\odot$, the Sun's radius, then

$$\Omega = 2\pi \{1 - [1 - (R_\odot/R)^2]^{1/2}\} .$$

Whenever $R \gtrsim 5 R_\odot$, $\Omega = \pi(R_\odot/R)^2$. The power re-radiated by the particle is equal to

$$4\pi s^2 \int_0^\infty Q_{abs} B(\lambda, T_g) \, d\lambda$$

where $B(\lambda, T_g)$ is Planck's function and T_g the temperature of the grain. Sublimation causes material to loose energy at the rate $4\pi s^2 L_s(T_g) \, dE/dt$, where dE/dt is the mass sublimation rate of the material at temperature T_g and $L_s(T_g)$ is the latent heat of sublimation at the same temperature. The equation giving T_g is

$$\frac{\Omega}{\pi} \int_0^\infty Q_{abs} F_\odot(\lambda) \, d\lambda = 4 \left[\int_0^\infty Q_{abs} B(\lambda, T_g) \, d\lambda + \frac{dE}{dt} L_s(T_g) \right]. \tag{1}$$

Obviously, it is easier to solve this equation for R rather than for T_g. Thus T_g is the parameter and the corresponding R is evaluated.

b) Numerical Solution

The two integrals in Eq. (1) were computed as follows: for each wavelength, a value of the complex refractive index $m(\lambda) = n(\lambda) - ik(\lambda)$ of the material is provided to evaluate the efficiency factor $Q_{abs}[m(\lambda), \lambda]$ using Mie theory (Lamy, 1974a) which is then multiplied by $F_\odot(\lambda)$ and $B(\lambda, T_g)$ separately. Integrations are carried out using the method of trapezoids with five steps to account in detail for the critical regions (absorption bands). The graph of the functions $Q_{abs}(m, \lambda) \, B(\lambda, T_g)$ versus λ (see below) will illustrate the accuracy of this method.

The domain of integration should cover the region where the bulk of the solar radiation is emitted as well as those where the particle's own infrared radiation takes place. Thus, they depend upon the minimum temperature considered for the particles, but are in fact limited by the knowledge of $m(\lambda)$. The following regions have been used:

quartz: $0.2 \, \mu - 40 \, \mu$,
obsidian: $0.2 \, \mu - 30 \, \mu$,
andesite: $0.2 \, \mu - 50 \, \mu$,
ice: $0.2 \, \mu - 100 \, \mu$,
iron: $0.25 \, \mu - 18 \, \mu$.

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Table 2. Heliocentric distances ($R_\odot$) versus temperature for interplanetary quartz (Q), obsidian (O) and andesite (A) grains of radius $s(\mu)$

$T(°K)$	$s=0.1$			$s=0.5$			$s=1$			$s=3$		
	Q	O	A	Q	O	A	Q	O	A	Q	O	A
200	24.5	47.7	192.7	24.2	52.7	245.9	22.5	51.1	247.9	22.9	46.8	218.4
400	5.93	9.6	40.8	5.9	10.7	52.2	5.78	10.8	53.5	6.2	12.2	54.8
600	3.23	5	21.4	3.24	5.5	27.3	3.22	5.6	28	3.45	6.6	29.1
800	2.31	3.5	14.7	2.32	3.9	18.6	2.32	3.9	18.9	2.46	4.6	19.2
1000	1.86	2.7	11.2	1.87	3.1	14.	1.87	3.1	14.	1.97	3.7	13.9
1200		2.3	9.		2.6	11.		2.6	10.9		3.	10.4
1400		2.	7.5		2.3	8.8		2.3	8.6		2.6	8.1
1600		1.8	6.2		2.	7.2		2.	7.		2.3	6.5

However, corrections have been applied to account for the contributions coming from regions beyond these upper limits as will be explained later. The narrower domain used for iron is due to the lack of data for the imaginary refractive index for longer wavelengths thus restricting its temperature to values larger than 300 °K approximately.

The details of the computation of the sublimation term $4\pi s^2 L_s\, dE/dt$ are presented in Part IV where it is shown that this term is negligible in the case of quartz, obsidian, andesite and iron.

c) Results

i) Quartz, Obsidian and Andesite

The temperature distribution for these three materials has been regrouped in Table 2. The temperature was allowed to reach 1600 °K to obtain an idea of the distribution near the Sun (remember that the variation of the index of refraction with temperature was not accounted for). In general, the heliocentric distance corresponding to a given temperature tends to increase with increasing grain radius. In other words, small particles are colder than large ones at a given heliocentric distance. Temperature distributions tend to be similar for $s=0.5$ and 1 μ and differ noticeably for $s=0.1$ and 3 μ. For these latter values, the scattering has changed somewhat since they clearly differ from the typical wavelengths of the solar spectrum. The increasing absorptive property of the three silicates from quartz to obsidian and andesite appears readily and is best illustrated in Fig. 1 in the case of grains 1 μ in radius. Also plotted are the results of Over (1958) and those of Swamy and Donn (1968). Over's idealization of uniform absorption in the infrared beyond about 3 μ led him to consider in fact a silicate slightly more absorbent than obsidian and not true quartz. The "impure quartz" of Swamy and Donn closely behaves like iron. Since Lamy (1974b) has shown that iron grains cannot get closer to the Sun than $25 R_\odot$ (as s decreases under
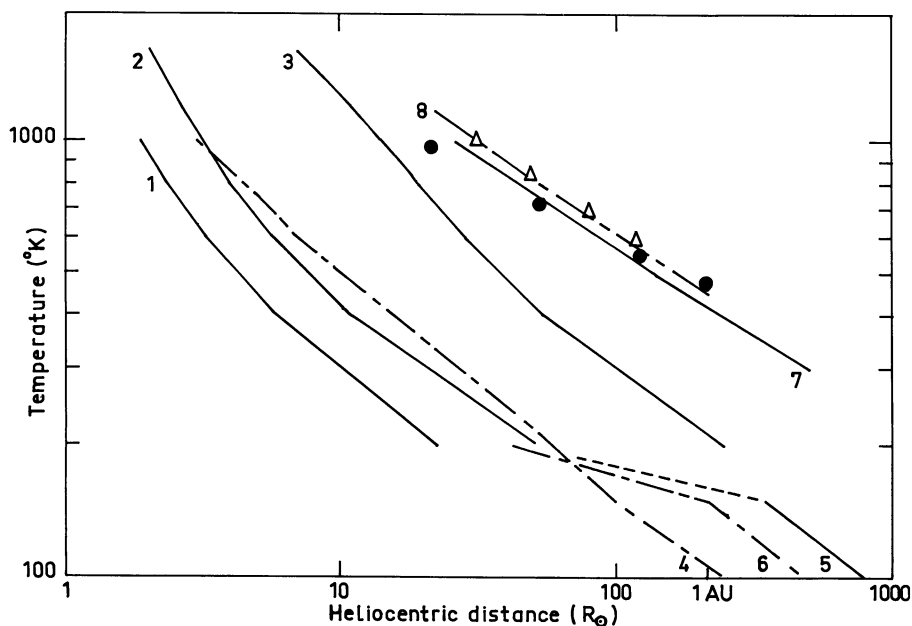

Fig. 1. Temperature distribution of interplanetary dust grains: quartz (1), obsidian (2), andesite (3), Over's result for quartz (4), Swamy and Donn's result for "impure quartz" (●), ice (5), Robey's result for ice (6), iron (7), Swamy and Donn's result for iron (8) and Becklin and Westphal's result for Comet 1965f (△). The results of Kaiser for small particles coincide exactly with curve 1

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sublimation, the radiation pressure force becomes relatively more important and the inward spiraling due to drags is stopped), the work of Swamy and Donn leads to the theoretical impossibility of the existence of a dusty component in the Corona. We see here that silicates can come very close to the Sun (2 to $3R_\odot$) with a temperature well below their melting point. Kaiser (1970)'s results for "small particles" are not plotted separately since they follow very closely those for quartz with $s = 1\,\mu$ (curve 1). The agreement is even better for smaller radii, i.e., $s = 0.1$ and $0.5\,\mu$. However grains with radius less than $1.5\,\mu$ were classified as "Rayleigh particles" by Kaiser and were assigned a slightly different temperature distribution. We think that Kaiser's four—part model—which avoids using Mie theory—is a good approximation leading to sound results but that it implies systematic errors. For instance, for a temperature of $400\,°K$, Kaiser's "Rayleigh particles" are located at $7R_\odot$ while our grains with $s \leqq 1\,\mu$ lie between 5.8 and $5.9\,R_\odot$.

ii) Ice

Results are presented in Table 3 where the heliocentric distance is given in astronomical units. The line $T = 200\,°K$ is of academic interest since the grains have then very short lifetimes (between 4 and 120 sec, see Part IV). For $T = 100\,°K$, the values of $\int Q_{abs}B(\lambda T)\,d\lambda$ have been corrected to take into account the contribution of wavelengths larger than $100\,\mu$ although this contribution is very small. At this temperature, the sublimation term $L_s\,dE/dt$ is so small that it can almost be neglected. At $T = 150\,°K$, it is no longer negligible and at $T = 200\,°K$, it has roughly the same order of magnitude as $\int Q_{abs}B(\lambda, T)\,d\lambda$. At $T = 250\,°K$, it becomes the preponderant term. The influence of the radius of particle is fairly pronounced, particularly when it becomes large ($s = 3\,\mu$). Once more, one has to be careful with these results as explained in Part II since the temperatures of 100 and $150\,°K$ are outside the range of validity for the index of refraction. On Fig. 1, we have plotted together with our results for $s = 1\,\mu$ those of Robey (1960) for "large" spheres ($s \gg \lambda$) of water-ice which absorbs $\gamma = 7\%$ of the incident radiant energy (see his Fig. 9). Both curves have the same broken shape explained by the strong effect of sublimation above $150\,°K$. Since Robey has considered several values of γ (between 1 and 100%), it is certainly possible to come to a close agreement by choosing the

most appropriate value of γ ($\simeq 30\%$). This, of course, is not very conclusive. Watson et al. (1963) studied the surface temperature of very large spheres ($s \simeq 1\,$km) of water-ice using Stefan's law. Even for their largest albedo, 60%, their results are larger by an average of $25\,°K$ for a given distance in the range studied here (0–4 a.u.). At constant temperatures, this corresponds to a discrepancy of 1 a.u. This shows how sensitive such results are and thus, the difficulty of this analysis. To get a better agreement, it would be necessary to consider an albedo larger than 60%, a fact also suggested by the large values (close to 100%) found for our small particles in the visible region. However, there is no real meaning in giving a further comparison, since the sizes of spheres considered, and therefore the methods, are completely different. Swamy and Donn (1968) studied the same problem but, since they consider "hot ice" only ($T \geqq 450\,°K$), no comparison is possible.

iii) Iron

Results are presented in Table 4 for $s = 0.5$ and $1\,\mu$ only due to computing problem. For $T = 300$ and $500\,°K$, the values of the integral $\int Q_{abs}B(\lambda, T)\,d\lambda$ (which was originally taken in the range 0.25–$18\,\mu$) have been corrected for the contribution corresponding to longer wavelengths, up to 40 and $30\,\mu$ respectively. Q_{abs} was assumed constant and equal to its value at $18\,\mu$ for this evaluation; this should lead to only very small errors since it concerns the tail of the energy spectrum. Sublimation is always negligible. The influence of the grain's radius, strong at low temperatures, tends to disappear with increasing temperatures and is negligible for $T \geqq 900\,°K$. As for the other materials, the remark concerning the validity of the results at high temperatures applies. Results for $s = 1\,\mu$ are plotted on Fig. 1 along with those of Swamy and Donn (1968) for the same radius. The good agreement, which is even better with our case $s = 0.5\,\mu$, is very surprising in view of the roughness and limited number of values for the imaginary index of refraction which they used. These latter values, borrowed from Gaustad (1963), have little to do with the ones selected for this study. This tends to prove that, contrary to silicates, the temperature distribution of iron is not very sensitive to the variation of the index of refraction with wavelength, a property which is probably true for any

Table 3. Heliocentric distance (a.u.) versus temperature for interplanetary water-ice grains of radius $s(\mu)$

$T(°K)$	$s = 0.1$	$s = 0.5$	$s = 1$	$s = 3$
100	3.7	3.85	3.7	2.8
150	1.52	1.65	1.6	1.16
200	0.07	0.165	0.22	0.28

Table 4. Heliocentric distance (a.u.) versus temperature for interplanetary iron grains of radius $s(\mu)$

$T(°K)$	$s = 0.5$	$s = 1$
300	3	2.37
500	0.77	0.64
700	0.36	0.284
900	0.165	0.16
1000	0.123	0.123

metals i.e., strongly absorbent materials. The color temperatures calculated from the 2.2 and 3.4 μ intensities of Comet 1965f obtained experimentally by Becklin and Westphal (1966) between 0.2 and 0.5 AU are plotted too on Fig. 1. Unfortunately, the agreement with our results is misleading. In order to understand this point, one has to remember the method used by Becklin and Westphal to find their measurements of intensities. They had to find a material with an emissivity such that the solar equilibrium temperature will agree with the color temperatures. Iron turns out to fit extremely well using the measurements at 2.2 and 3.4 μ (this is why the corresponding temperature distribution has been plotted) but rather poorly when calculating the color temperature from the 3.4 and 10 μ intensities. Of course, there is the possibility of a straightforward comparison between their experimental intensities measured at four wavelengths and the theoretical intensities for iron which are given by $Q_{abs}B(\lambda, T)/\pi$. No agreement is possible because the former quantities turn out to be much too low. This fact is readily explained by the assumption made by Becklin and Westphal that all the intensity at 1.65 μ was scattered sunlight, the other points being corrected accordingly. However, for temperatures that iron is expected to reach between 0.5 and 0.2 AU ($T \gtrsim 550$ °K), the thermal emission at 1.65 μ is of the same order of magnitude as that at 2.2 μ. This has already been pointed out by Swamy and Donn (1968) together with the necessity of correcting all the observations for molecular emission. We therefore agree with them that no meaningful comparison will be possible until the observations are correctly reduced. Bandermann's (1969) formula

$$T(R) = 418 \, R^{-0.27} \, °K \qquad (R \text{ in a.u.})$$

supposedly derived from the observations of Becklin and Westphal, should be considered erroneous since it does not at all fit their results.

iv) Emission Spectra

For all values of radius and temperature appearing in Tables 2, 3 and 4, the intensity spectrum radiated by the corresponding grain has been obtained in the spectral regions introduced in § IIIb. This is the fonction $Q_{abs}B(\lambda, T)/\pi$ (expressed in W cm^{-2} μ^{-1} sterad^{-1}) of λ. It is of course impossible to present here such an enormous amount of data. We thus limit ourselves to the emission spectra of 1 μ spheres of obsidian and andesite both at 400 °K, of iron at 500 °K (Fig. 2) and of ice at 150 °K (Fig. 3). In the case of dielectrics, they reflect the absorption band structure weighted, however, by the Planck function at the appropriate temperature; the shape of the emission spectra are therefore strongly dependent on temperature, i.e., on whether or not the maximum of the Planck function $B(\lambda, T)$ coincides
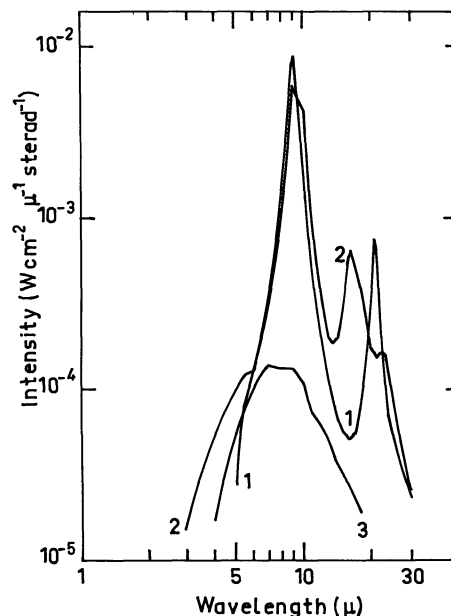

Fig. 2. The emission spectra of 1 μ spheres of obsidian (1) and andesite (2) both at 400 °K and of iron (3) at 500 °K
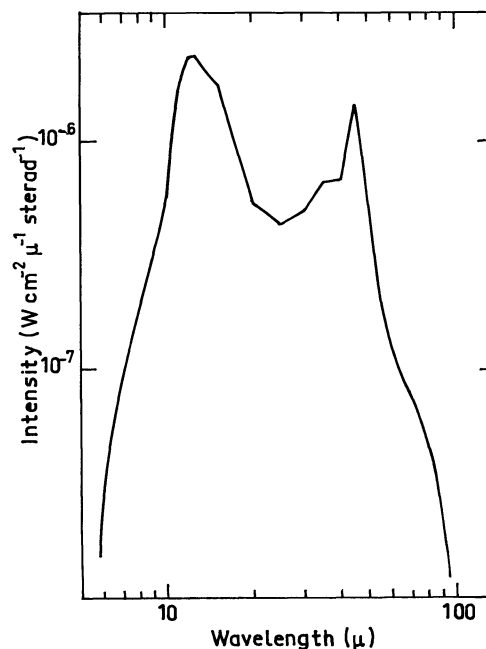

Fig. 3. The emission spectrum of a 1 μ sphere of water-ice at 150 °K

with an absorption band. As iron possesses a smooth absorption spectrum, its emission spectrum closely ressembles that of a black-body having a shape fairly independent of temperature.

IV. Sublimation of Interplanetary Dust Grains

The purpose of this part is twofold: first, to present the method of evaluating the sublimation term $L_s \, dE/dt$ in Eq. (1) and as a direct consequence, to derive the

erosion rate of the materials considered here so as to obtain the lifetimes of interplanetary dust grains as controlled by sublimation. The value dE/dt is a characteristic physical property of any solid phase that depends only on the surface temperature of that solid and is equal to $\delta|ds/dt|$ where δ is the density of the material. Therefore a grain of initial radius s_0 loses mass at the rate

$$\frac{dm}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi s^3 \delta\right) = 4\pi s^2 \frac{dE}{dt}$$

and has a maximum lifetime

$$= \frac{s_0}{|ds/dt|} = \frac{s_0 \delta}{dE/dt}.$$

a) The Mass Sublimation Rate

Delsemme and Swings (1952), Estermann (1955) and Delsemme (1966), among others, have discussed at considerable length how to evaluate the rate of sublimation. Briefly, this is done by considering a dynamic equilibrium between evaporation and condensation so that the vapor pressure p is maintained in the gas phase. The maximum rate of evaporation for a surface exposed to a perfect vacuum is thus equal to the number of molecules striking a unit surface in unit time. This term is given in the kinetic theory of gases as

$$Z = pN(2\pi MRT)^{-1/2}$$

[Estermann, Eq. (7-1), p. 743] where p is the vapor pressure, N is Avogadro's number, M is the molecular or atomic mass, R is the gas constant and T is the temperature. It will be assumed that the temperature T of the vapor is the same as the temperature of the particle T_g. The difference between the two will always be negligible, as discussed by Gaustad (1963). Experiments show that, in the case of evaporation from crystalline surfaces, the real rate is always smaller than its maximum theoretical value. Therefore, the above theoretical value must be multiplied by a factor $\alpha(\alpha < 1)$ known as the "accommodation coefficient".
Now the radius of the grain decreases at the rate

$$\left|\frac{ds}{dt}\right| = \frac{MZ}{N\delta} = \frac{\alpha}{\delta}p(M/2\pi RT)^{1/2}$$

while the mass sublimation rate amounts to

$$\frac{dE}{dt} = \alpha p(M/2\pi RT)^{1/2}.$$

If p is expressed in Tor (mm of mercury), M is in units of molecular or atomic mass and T is in degrees Kelvin, then E is given in gm s^{-1} cm^{-2} by

$$\frac{dE}{dt} = 5.83 \times 10^{-2}\,\alpha p\sqrt{(M/T)}$$

which is similar to Eq. (1) of Delsemme and Swings (1952), for $\alpha = 1$. Because is generally close to 0.7, the value adopted by Delsemme (1965), we have retained this value. Finally,

$$\frac{dE}{dt} = 4.08 \times 10^{-2}\,p\sqrt{(M/T)}\ \text{gm s}^{-1}\ \text{cm}^{-2}.$$

Thus the vapor pressure p is of prime importance in all these computations. The discrepancies often found between formulae has led to the following discussions.

i) Quartz, Obsidian and Andesite

Considerable controversy has taken place in the past for the case of quartz. Refined experiments have now shown that the theoretical formula of Eucken often used in astrophysics is in grave error since a molecule of SiO_2 is not directly evaporated but first dissociated in SiO and $1/2\,O_2$. SiO_2 (gas) is also present, but in a very small quantity.
Brewer and Mastik (1951) and Porter et al. (1955) have shown that the partial vapor pressure of SiO_2 and O_2 are smaller than that of SiO by at least two orders of magnitude, so that the vapor pressure law is primarily governed by SiO. Schik (1960) gave a critical review of all experimental data and compared them to theoretical results. He then proposed two formulae:

$$\log p_{SiO}(\text{Tor}) = 10.876 - \frac{25\,265.2}{T} \qquad T > 2000\,^{\circ}\text{K}$$

$$\log p_{SiO}(\text{Tor}) = 11.445 - \frac{26\,335}{T} \qquad T < 2000\,^{\circ}\text{K} \tag{2}$$

where the second one is of interest in this study since temperatures considered are less than 2000 °K. For the purpose of comparison, we recall the Eucken-Urey formula derived for 2000 °K:

$$\log p(\text{Tor}) = 10.531 - \frac{22\,200}{T}$$

and the vapor pressure of tektite glass evaluated experimentally by Centolanzi and Chapman (1966) between 1700 and 3000 °K:

$$\log p(\text{Tor}) = 10.915 - \frac{24\,928.3}{T}. \tag{3}$$

Figure 4 in which the three equations are plotted in the temperature range 1000–2000 °K, shows the large discrepancy between the Eucken-Urey equation and the one given by Schik (two orders of magnitude) and the good agreement between the latter and the formula of Centolanzi and Chapman. In the range of validity of this last one (1700–3000 °K), the disagreement is by no more than a factor of two. This can be readily explained by the fact that the tektite glass used by Centolanzi and Chapman is not pure SiO_2 but
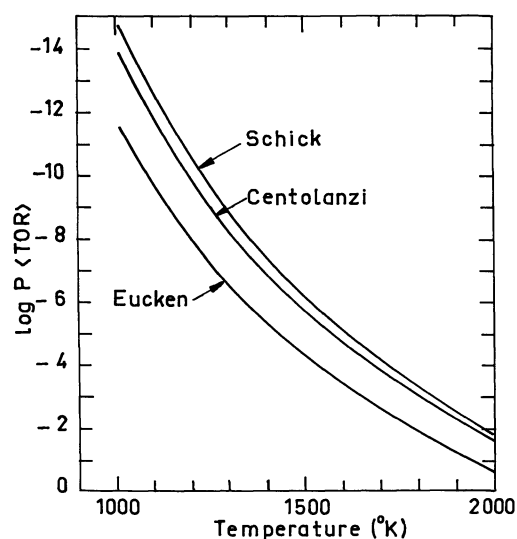
Fig. 4. The vapor pressure of quartz

Table 5. Characteristic quantities for sublimation of silicates

T (°K)	p (Tor)	$\dfrac{dE}{dt}$ (g cm^{-2} s^{-1})	τ (s) ($s_0 = 1\mu$)
Quartz			
900	1.6×10^{-18}	1.7×10^{-20}	1.6×10^{16}
1000	1.3×10^{-15}	1.3×10^{-17}	2.1×10^{13}
1100	3.2×10^{-13}	3×10^{-15}	8.7×10^{10}
1200	3.2×10^{-11}	2.9×10^{-13}	9.2×10^{8}
1300	1.5×10^{-9}	1.35×10^{-11}	2×10^{7}
1400	4.3×10^{-8}	3.64×10^{-10}	7.3×10^{5}
1500	7.7×10^{-7}	6.3×10^{-9}	4.2×10^{4}
1600	9.7×10^{-6}	7.7×10^{-8}	3.5×10^{3}
Obsidian			
1000	1×10^{-14}	1.1×10^{-16}	2.2×10^{12}
1200	1.4×10^{-10}	9.7×10^{-13}	2.4×10^{8}
1400	1.3×10^{-7}	9×10^{-10}	2.6×10^{5}
1600	2.2×10^{-5}	8.4×10^{-8}	2.8×10^{3}

contains non-negligible amounts of Al_2O_3, FeO, MgO and, possibly, other oxides. It is of course Eq. (2) which applies for quartz while Eq. (3) seems more appropriate to obsidian and andesite owing to their composition.

ii) Ice

The International Critical Tables (1926–1930) gives a highly reliable formula

$$\log p(\text{Tor}) = -\frac{2445.6}{T} + 8.23 \log T - 0.01677\,T$$
$$+ 1.205 \times 10^{-5} T^2 - 6.757$$

valid in the range $-100°$ C to $0°$ C (173.16–273.16 °K). At lower temperatures, one generally derives a formula from classical thermodynamic arguments with simplifying assumptions. In this manner, Greenberg (1968) derived a formula good for $T < 100$ °K with an accuracy no better than a factor of 3 to 5:

$$\log p(\text{Tor}) = -\frac{2480}{T} + 4 \log T + 0.935\,.$$

Use of statistical thermodynamics drastically improves the accuracy and Kelly (1935) obtained:

$$\log p(\text{Tor}) = -\frac{2461}{T} + 3.857 \log T + 3.41 \times 10^{-3}\,T$$
$$+ 4.875 \times 10^{-8} T^2 + 1.207\,.$$

iii) Iron

Urey's (1952) classical formula, good around 2000 °K, does not apply to our problem since the temperatures of interest are much lower. Hultgren's (1963) compilation in the range 298.15 to 3500 °K (p. 109) and the formula (which we retained)

$$\log p(\text{Tor}) = -\frac{21\,080}{T} + 16.89 - 2.14 \log T$$

Table 6. Characteristic quantities for sublimation of ice

T (°K)	p (Tor)	$\dfrac{dE}{dt}$ (g cm^{-2} s^{-1})	τ (s) ($s_0 = 1\,\mu$)
50	5.1×10^{-42}	1.3×10^{-43}	7.3×10^{38}
80	1.1×10^{-22}	2.2×10^{-24}	4.2×10^{19}
100	4.5×10^{-16}	7.8×10^{-18}	1.2×10^{13}
150	5.1×10^{-7}	7.2×10^{-9}	1.3×10^{4}
180	4.8×10^{-5}	6.8×10^{-3}	1.5×10^{2}
200	1.2×10^{-3}	1.6×10^{-1}	6.2
250	5.7×10^{-1}	6.9×10^{1}	< 1
273.2	4.6	5.3×10^{2}	< 1

Table 7. Characteristic quantities for sublimation of iron

T (°K)	p (Tor)	$\dfrac{dE}{dt}$ (g cm^{-2} s^{-1})	τ (s) ($s_0 = 1\,\mu$)
900	1.4×10^{-13}	1.4×10^{-15}	5.5×10^{11}
1100	1.7×10^{-9}	1.5×10^{-11}	5.1×10^{7}
1300	1×10^{-6}	8.7×10^{-9}	9×10^{4}
1500	1.1×10^{-4}	8.7×10^{-7}	9×10^{2}

of Smithells (1967, p. 262) valid in the interval 900 to 1812 °K, shows good agreement and give one faith in their results. The discrepancy, at most a factor of 2.5, tends to level off with increasing temperature and is of no importance in view of the accuracy needed in this study. Smithell's formula is preferred owing to its flexibility.

Numerical results for the vapor pressure and the mass sublimation rates are presented in Tables 5, 6 and 7. Those for obsidian and andesite are so close that they have been regrouped. Included there are the lifetimes τ of grains having an initial radius $s_0 = 1\,\mu$, a question that we now discuss.

b) Lifetimes of Interplanetary Grains

A typical grain of quartz at 1000 °K, for instance, will last 2×10^{13} s $\simeq 2 \times 10^5$ years. Belton (1966) carried out a similar evaluation of τ for quartz with $s_0 = 1\,\mu$ but he does not give any of the details of his method. His values for $T = 500$, 1000 and 1500 °K are obviously wrong; it may be that he has a misprint in the sign of the exponent of 10. If this is so, his results after correction read $\tau = 4 \times 10^{+7}$ s for $T = 1000$ °K (compared to 2×10^{13} s in this study) and $\tau = 5 \times 10^{+1}$ s for $T = 1500$ °K (compared to 4.2×10^4 s). Obsidian (and andesite) grains have about the same lifetimes as quartz as witnessed by the typical values given in Table 5.

Ice grains experience a very fast destruction as soon as their temperature reaches approximately 150 °K (see Table 6). A comparison with the values of Watson *et al.* (1961) for the sublimation rate for temperatures between 72 °K and 122 °K shows a good agreement, although they used the International Critical Tables formula for the vapor pressure; the resulting discrepancy is minor even in this low temperature range. Gaustad (1963) found a lifetime of 9×10^7 s for a particle of $0.2\,\mu$ radius at 125 °K. This corresponds to 4.5×10^8 s for a radius of $1\,\mu$, compared to 2.3×10^8 s according to the present method. Hence, the agreement is very good. By writing that complete evaporation occurs when the vapor pressure exceeds the stoichiometric pressure (computed from the abundance of elements), he obtained the result that ice will have sublimated before the temperature rises above 200 °K. Our conclusion agrees with this but also shows that sublimation becomes extremely effective at 150 °K ($\tau = 20$ hrs for $s_0 = 1\,\mu$).

The behavior of iron is similar to quartz (see Table 7) in that high temperatures are required to get a substantial loss rate. At 1100 °K, the lifetime of a $1\,\mu$ particle is 1.6 years.

It was felt that the above results do not give a sufficiently clear picture of the evolution of our grains in interplanetary space as controlled by sublimation. At a given heliocentric distance R, a grain reaches a certain temperature leading to a lifetime τ. The ratio of τ to the keplerian period corresponding to a circular orbit of radius R—i.e., the number of revolutions for a grain with initial radius s_0—is proposed as a more tangible expression of the lifetime. These results are plotted in Fig. 5 for all five materials with $s_0 = 1\,\mu$. As τ is proportional to s_0, it is a simple matter—by translation along the vertical axis—to extend these results to any value of the grains' initial radius. The location of the onsets of rapid sublimation appear clearly on Fig. 5: $1.5\,R_\odot$ for quartz, $2.2\,R_\odot$ for obsidian, $9.5\,R_\odot$ for andesite, $11\,R_\odot$ for iron and 2.8 a.u. for ice. "Evaporation" rates for stone and iron have been given by Singer and Bandermann (1967) without any
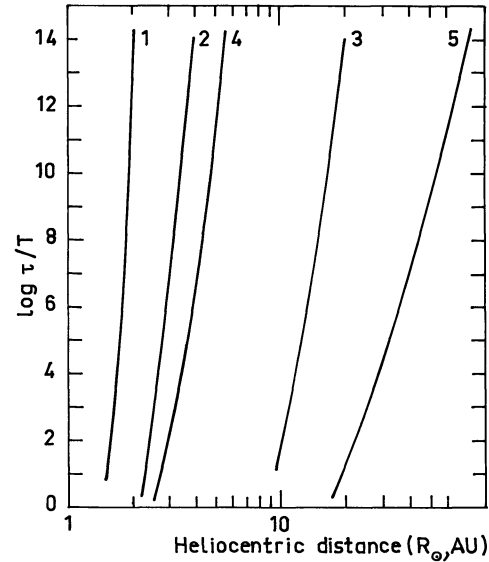

Fig. 5. Lifetimes of interplanetary dust grains of initial radius $1\,\mu$, expressed in number of keplerian revolutions. The unit of heliocentric distance is $R_\odot$ for quartz (1), obsidian (2), andesite (3) and iron (5), and a.u. for ice (4)

details; they agree very poorly with ours. At 0.1 a.u., for instance, they found 1.5×10^5 Å yr^{-1} for stone and 7.3×10^1 Å yr^{-1} for iron while our computations yield 6.2×10^{-11} (in the most favorable case, that of andesite) and 4.7×10^3 Å yr^{-1} respectively. Not knowing any of their assumptions prevents any further discussion. However, it is most surprising that their "evaporation" rate for stone is much larger than that of iron.

c) The Term $L_s \, dE/dt$

For the sake of completeness of the present study, we briefly mention how the latent heat of sublimation was obtained. As the term $L_s \, dE/dt$ is often negligible, L_s does not need to be known accurately. The method based on the Clausius-Clapeyron equation was used taking dp/dT by differentiation of the vapor pressure expressions given in § IVa and making the usual appropriate assumptions. We retained the following values expressed in kcal (g · mole)$^{-1}$: 135 for quartz, 114 for obsidian and andesite, 12.2 for ice and 110 for iron approximately. The term $L_s \, dE/dt$ turns out to be important only in the case of ice at temperatures around and above 150 °K.

V. Radiation Pressure for Interplanetary Dust Grains

a) Equation for the Radiation Pressure Force

The radial force due to solar radiation pressure is:

$$F_r = \frac{s^2 \Omega}{c} \int_0^\infty Q_{pr}(m, \lambda) \, F_\odot(\lambda) \, d\lambda$$

where c is the velocity of light and $Q_{pr}(m, \lambda)$ the efficiency factor for radiation pressure as given by the Mie-Debye theory. Far from the Sun, F_r becomes

$$F_r = \pi \frac{s^2}{c} \left(\frac{R_\odot}{R} \right)^2 \int_0^\infty Q_{pr} F_\odot(\lambda) \, d\lambda .$$

The ratio $\beta = F_r/F_g$ where F_g stands for the Sun's gravitational attraction offers a convenient measure of F_r. It is given by

$$\beta = \frac{9.1265 \times 10^{-5}}{s\delta} \int_0^\infty Q_{pr} F_\odot(\lambda) \, d\lambda$$

(s in μ, δ in gm cm^{-3} and the integral in W cm^{-2}), an expression valid for $R \gtrsim 4R_\odot$. Under this restriction, β is independent of R. The integral contained in the expression for β was calculated in the same manner as those for absorption (see § IIIb). As an interesting consequence, it has been possible to verify that the bulk of the contribution to the integral for radiation pressure comes indeed from the region 0.2 to 0.4 μ—as pointed out by several authors (Gindelis et al., 1969, for instance)—since we carried out all integrations on wider domains. To be even more explicit, 50% of the value of this integral comes from the region 0.2 to 0.6 μ while 35% of the solar energy is contained in this spectral interval. Notice the importance of the solar spectrum in this domain.

b) Results

For quartz, obsidian, andesite and ice, β has been evaluated for eight values of the radius $s = 0.05$, 0.1, 0.2, 0.3, 0.4, 0.5, 1 and 3 μ, and for $s = 0.025$, 0.05, 0.1, 0.15, 0.2, 0.5, 1 μ in the case of iron. β is plotted versus s on Fig. 6, and presents the following features:
 i) All three curves have similar shapes,
 ii) The graphs for quartz, andesite, obsidian and ice are close to each other, the maximum of β being 0.73, 0.59, 0.57 and 0.61 respectively. Thus the curve for quartz peaks some what more, a fact which tends to show that impurities in silicates cause β to decrease in the interval 0.1 to 0.5 μ. Beyond 0.5 μ, they have little effect. This phenomenon tends to level off when the absorption of the substances increases (at least, in a reasonable range) as witnessed by the extreme closeness of the curves of andesite and obsidian. The maxima found here may be overestimated slightly because of our model of solar spectrum but this is not important since we are assured that β remains smaller than one for spherical particles of quartz, obsidian, andesite and ice, of any radius having the optical properties assumed here. It is expected that slight changes in these optical properties should not affect this conclusion.
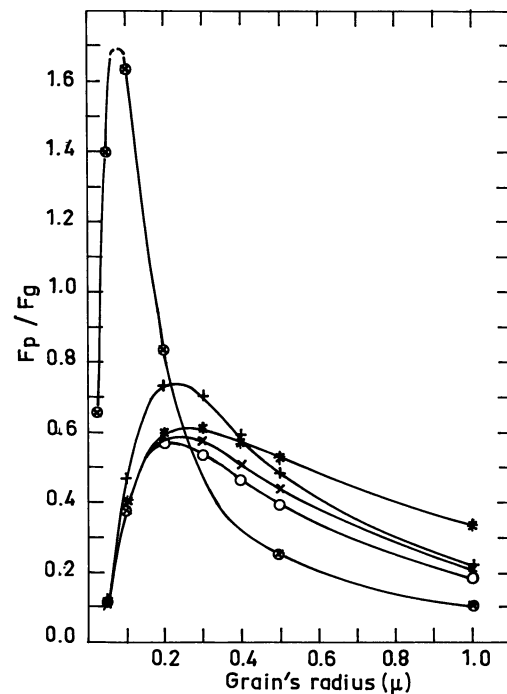 iii) The curve for iron presents a sharp peak reaching about 1.63. Consequently, particles having a radius

Fig. 6. The ratio F_p/F_g of the solar radiation pressure force to the Sun gravitational force for spheres of quartz (+), obsidian (○), andesite (×), ice (✻) and iron (⊗) as a function of their radii

such that $0.04 \, \mu \lesssim s \lesssim 0.17 \, \mu$ are subjected to a force due to radiation pressure which is larger than the gravitational attraction of the sun ($F_p/F_g > 1$). It is clearly seen that a slight error in β will not much affect the boundary values given in the preceding inequality since the peak is very sharp.

Our radiation pressure results are now compared to what seem to be the best results available in the literature. For quartz, whatever the approximations, there is a general agreement that F_p/F_g remains smaller than 1 for particles of all radii. However, there are differences concerning its maximum value: Shapiro et al. (1966) agree closely with our value of 0.73, while Gindilis et al. (1969) obtained 0.6 with a density of 2.5. Hence, correcting for the density, this value becomes 0.57. For ice, Aller et al. (1967) performed a similar evaluation for "pure ice" and two models of "dirty ice" keeping the various indices of refraction constant for each of three cases. They found that ice must be quite dirty (approximately $k \gtrsim 0.04$ over the whole absorption spectrum) in order that β exceed 1. Otherwise, their result agree with ours, β remaining smaller than 1 for all radii. For iron, Shapiro et al. (1966) and Gindilis et al. (1969) found that the curve of β versus s crosses the $s = 0$ axis at values larger than 1 (1.4 and 1.9, respectively) which means that all particles of radius smaller than $\simeq 0.17 \, \mu$ are ejected. We disagree on this point, since our curve is similar in shape to the others, it does not intersect the ordinate axis and, thus, F_p/F_g becomes smaller than 1 for $s \lesssim 0.04 \, \mu$. Indeed, we may

expect the radiation force to become very small for particles of small radii since such particles will not affect much the transmission of light; i.e., they will be transparent. But we agree with other researches that $s \simeq 0.17$ when $F_p/F_g = 1$. This is not surprising since an error in the maximum value of the ratio does not particularly affect this result, as pointed out before. As a matter of fact, our maximum value is not very well defined because of the sharpness of the peak and differs somewhat from those given in the two articles cited. But this point is of little importance.

VI. Conclusions

There is no need to summarize here the main results of this study as they are well illustrated in Figs. 1, 5 and 6. However, their direct consequence on the behavior of interplanetary dust calls for further comment. First, we have shown that silicate grains can come very close to the Sun with a reasonable temperature giving them a fairly long lifetime. This confirms the theoretical possibility of existence of a dusty component—made of real-life materials—of the solar corona, i.e., the "outer corona" familiar to the observers. Well-defined dust free zones are found, whose location depends strongly upon the optical properties of the grains' material. Although this gives a fairly good picture of the situation in the vicinity of the Sun, one should keep in mind that these zones are static. The dynamics of circum-solar grains was investigated by the author (Lamy, 1974b) and dynamical dust free zones were found. Finally, the familiar picture showing all grains expelled sooner or later from the solar system (Belton, 1966, 1967; Kaiser, 1970) should be considered incorrect for silicate and ice grains, since the corresponding ratios F_p/F_g remain less than 1 for all grain's radii. The reader is referred to another article (Lamy, 1974b) for further comment on this problem.

Note added in proof: The work of T. Mukai and S. Mukai (1973, *Publ. Astron. Soc. Japan* **25**, 481) to which the author has just been referred provides an interesting basis of comparison for the cases of quartz and ice.

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Ph. Lamy
Laboratoire d'Astronomie Spatiale
Les Trois Lucs
F–13012 Marseille, France