Astro/EPS C12 (2008) – Mike Wong
Problem Set 1 Solution Guide

1 Surface gravity

Problem 1A

First find a formula for g:

$$F = m_{\text{object}}\ g$$
$$F = \frac{G\ m_{\text{planet}}\ m_{\text{object}}}{r^2}$$

$$m_{\text{object}}\ g = \frac{G\ m_{\text{planet}}\ m_{\text{object}}}{r^2}$$
$$g = \frac{G\ m_{\text{planet}}}{r^2}$$

For the last column of the table, just divide by the Earth's surface gravitational acceleration $g_{\oplus} = 9.8\ \text{m}\,\text{sec}^{-2}$.

As an example, here is the calculation for the International Space Station. The other parts of the problem are done the same way. Use the distance to the Earth's center (350 km + the Earth's radius) and the mass of the Earth, because the space station itself is too tiny to exert much of a gravitational pull on anything.

$$r = r_{\oplus} + 350\ \text{km} = 6370\ \text{km} + 350\ \text{km} = 6720\ \text{km} \times \frac{1000\ \text{m}}{1\ \text{km}} = 6.72 \times 10^6\ \text{m}$$
$$m_{\text{planet}} = 5.97 \times 10^{24}\ \text{kg}$$
$$g = \frac{G\ m_{\text{planet}}}{r^2} = \frac{6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\,\text{sec}^{-2}\ \ 5.97 \times 10^{24}\ \text{kg}}{(6.72 \times 10^6)^2\ \text{m}^2} = 8.82\ \text{m}\,\text{sec}^{-2}$$
$$\%\ \text{of Earth surface gravity} = g/g_{\oplus} = 8.82\ \text{m}\,\text{sec}^{-2}\ /\ 9.8\ \text{m}\,\text{sec}^{-2} = 90\%$$

Location	Distance to center of planet/object (m)	Mass of planet/object (kg)	Gravitational acceleration (m s^{-2})	Gravitational acceleration (% of Earth surface gravity)
Surface of the Moon	1.74×10^6	7.35×10^{22}	1.6	17%
Surface of Pluto	1.18×10^6 (p.75)	1.3×10^{22}	0.62	6.4%
Surface of Ceres	4.57×10^5	9.4×10^{20}	0.3	3%
Surface of Enchilladus	5.4×10^5	1.1×10^{21}	0.25	2.6%
International Space Station orbiting 350 km above Earth's surface	6.7×10^6	5.97×10^{24}	8.8	90%

Problem 1B

"Weight is a measure of gravitational force" means we can use the F from part 1A as the weight. Then:

$$F_\oplus = m_{\text{object}}\ g_\oplus$$
$$F_{\text{Pluto}} = m_{\text{object}}\ g_{\text{Pluto}}$$

$$\frac{F_{\text{Pluto}}}{F_\oplus} = \frac{m_{\text{object}}\ g_{\text{Pluto}}}{m_{\text{object}}\ g_\oplus}$$
$$F_{\text{Pluto}} = F_\oplus \frac{g_{\text{Pluto}}}{g_\oplus} = 150 \text{ lbs} \times 6.4\% = 9.53 \text{ lbs}$$

Problem 1C

Solve the equation for g:

$$g = F/m = 150 \text{ lbs}/150 \text{ lbs} = 1$$

So the answer is just $g = 1$, which is valid only at the Earth's surface. So g is actually a convenient unit of acceleration, used for measuring accelerations felt by astronauts during launch, by roller-coaster riders, and by football tacklers.

Problem 1D

The astronauts experience a zero-gravity environment because they are in a kind of free-fall. The 0.9 g acceleration at the space station is what keeps it in orbit around the Earth. Astronauts train for zero-g environments by riding in an airplane called the "vomit comet," which goes up high, then dives down. During the descent, passengers feel weightless because they're free-falling.

2 Density

Problem 2A

Use the formula for density, $\rho = m/V$. But first calculate the volume:

$$V = 4/3\ \pi\ r^3 = 4/3\ \pi\ 540000^3\ \text{m}^3 = 6.6 \times 10^{17}\ \text{m}^3$$

$$\rho = \frac{m}{V} = \frac{1.1 \times 10^{21}\ \text{kg}}{6.6 \times 10^{17}\ \text{m}^3} = 1670\ \text{kg m}^{-3}$$

$$1670\ \text{kg m}^{-3} \times \frac{1000\ \text{g}}{1\ \text{kg}} \times \frac{1\ \text{m}^3}{100^3\ \text{cm}^3} = 1.67\ \text{g cm}^3$$

Problem 2B

If Enchilladus were made out of rock + metal, then the density would be somewhere between the density of rock and the density of metal, depending on how much of each were present. So if this were true, then the density would have to be between 2.7 and 7.9 g cm^3. Enchilladus has a density too low for it to be composed of rock and metal.

Problem 2C

To test the scientist's claim, calculate the mass of Enchilladus using the given structure. The core radius is just 540 km - 220 km = 320 km. First get the core volume:

$$V_{\text{core}} = 4/3\ \pi\ r_{\text{core}}^3 = 4/3\ \pi\ (320000\ \text{m})^3 = 1.4 \times 10^{17}\ \text{m}^3$$

The shell volume is just the total volume (see 2A) minus the core volume:

$$V_{\text{shell}} = V - V_{\text{core}} = 6.6 \times 10^{17}\ \text{m}^3 - 1.4 \times 10^{17}\ \text{m}^3 = 5.2 \times 10^{17}\ \text{m}^3$$

Now we can calculate the total mass for this structure:

$$m = m_{\text{core}}\ +\ m_{\text{shell}} = \rho_{\text{rock}} V_{\text{core}}\ +\ \rho_{\text{ice}} V_{\text{shell}}$$

$$m = (2700\ \text{kg m}^{-3})(1.4 \times 10^{17}\ \text{m}^3) + (1000\ \text{kg m}^{-3})(5.2 \times 10^{17}\ \text{m}^3) = 8.9 \times 10^{20}\ \text{kg}$$

This is lower than the observed mass of Enchilladus, so it cannot be correct. We would need to have more high density material (like rock) and less low density material, to get a larger total mass. This would be the case if the ice shell were thinner and the rock core were larger.