Astro/EPS C12 (2008) – Mike Wong
Problem Set 2 Solution Guide

1 Variation of insolation

Problem 1A

Using the picture, you can find simple formulae for r_{ap} and r_{peri}:

$$r_{\mathrm{ap}} = a + c$$
$$r_{\mathrm{peri}} = a - c$$

Then use the equation we're given, which relates c, a, and e, to eliminate c from the formulae for r_{ap} and r_{peri}:

$$c = e\ a$$

$$r_{\mathrm{ap}} = a + c = a + e\ a = (1 + e)\ a$$
$$r_{\mathrm{peri}} = a - c = a - e\ a = (1 - e)\ a$$

Problems 1B and 1C
Here is the full calculation for these parts of problem 1, for Comet Halley. Calculations for the other bodies are very similar.

$$r_{\mathrm{peri}} = (1 - e)\ a = (1 - 0.967)\ 17.9\ \mathrm{AU} = 0.59\ \mathrm{AU}$$
$$r_{\mathrm{ap}} = (1 + e)\ a = (1 + 0.967)\ 17.9\ \mathrm{AU} = 35.2\ \mathrm{AU}$$

$$F_{\mathrm{peri}} = F_0\ (\frac{r_0^2}{r_{\mathrm{peri}}^2}) = 1370\ \mathrm{W\ m^{-2}}\ (\frac{1^2\ \mathrm{AU}^2}{0.59^2\ \mathrm{AU}^2}) = 3930\ \mathrm{W\ m^{-2}}$$
$$F_{\mathrm{ap}} = F_0\ (\frac{r_0^2}{r_{\mathrm{ap}}^2}) = 1370\ \mathrm{W\ m^{-2}}\ (\frac{1^2\ \mathrm{AU}^2}{35.2^2\ \mathrm{AU}^2}) = 1.11\ \mathrm{W\ m^{-2}}$$

$$\text{Variation} = \frac{F_{\mathrm{peri}}}{F_{\mathrm{ap}}} = \frac{3930\ \mathrm{W\ m^{-2}}}{1.1\ \mathrm{W\ m^{-2}}} = 3540 = 354,000\%$$

Since I put F_{peri} on the top, the variation tells us how much greater the perihelion insolation is than the aphelion insolation. If I had put the aphelion insolation on top, then the variation would be 0.028%, telling us that the aphelion insolation is less than a tenth of a percent of the perihelion insolation.

Actually, I should have defined the annual variation better. In normal speech, we would probably subtract 100% from each of the values in the table below. For instance, you would probably say that the Earth has a 7% annual insolation variation, not 107%.

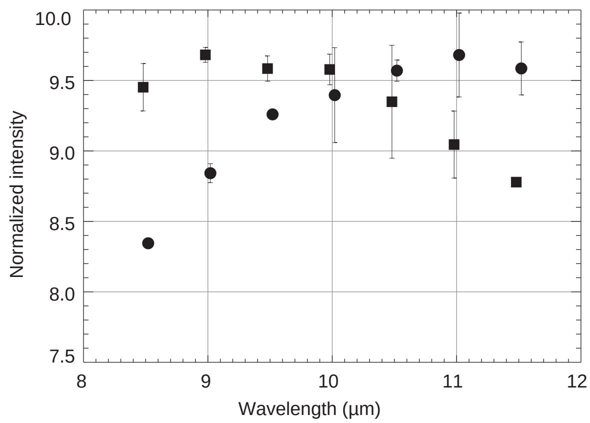
Object	Eccentricity e	Semimajor axis (AU)	Perihelion distance (AU)	Aphelion distance (AU)	Perihelion insolation ($W\ m^{-2}$)	Aphelion insolation ($W\ m^{-2}$)	Annual variation in insolation (%)
Mercury	0.206	0.387	0.31	0.47	14500	6290	231%
Earth	0.017	1	0.98	1.01	1420	1320	107%
Mars	0.093	1.52	1.38	1.66	721	496	145%
Jupiter	0.048	5.20	4.95	5.45	55.9	46.1	121%
Pluto	0.249	39.5	29.7	49.3	1.56	0.56	277%
Comet Halley	0.967	17.9	0.59	35.2	3930	1.11	355000%

Problem 1D

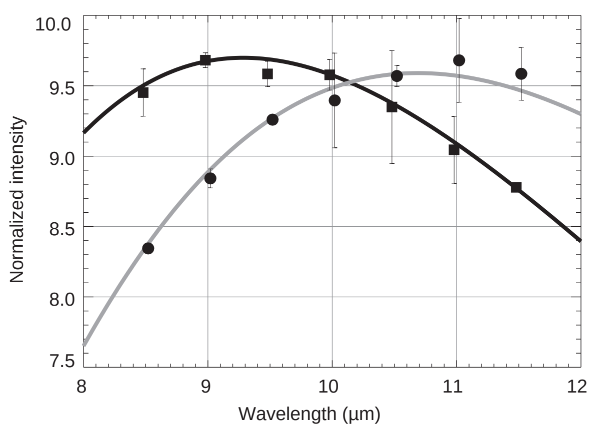
Kepler's second law describes how an object moves slower along its elliptical orbit when it is farther from the Sun, and moves faster when it's closer to the Sun. So although a comet receives very intense sunlight near perihelion, it spends much more of its time far from the Sun, where the insolation is small and temperatures stay cold enough to keep the volatiles frozen.

2 Measuring temperature

Problem 2A



Problem 2B
Answers may vary, depending on the values of λ_{max} found. To find λ_{max}, try to draw a smooth curve that fits within one errorbar of each data point. The peak of this curve is λ_{max}. Simply connecting the dots will get you close to the right answer, but is not the best approach. Connecting the dots is equivalent to assuming that each intensity measurement is exactly correct, but it is not. It is only correct to within the uncertainty. The statistics are actually a bit more complicated, but this is the principal behind interpreting data AND their uncertainties, as discussed (for example) with Fig. 2.4 in the book.



The smooth curves I've drawn are the thermal spectra of each cat. Based on these, I would find $\lambda_{\mathrm{max}} = 9.3\ \mu$m for Cat 1, and $10.7\ \mu$m for Cat 2. Then:

$$\lambda_{\max}\, T = 2900\ \mu\text{m K}$$
$$T = 2900\ \mu\text{m K} \;/\; \lambda_{\max}$$

$$T_1 = 2900\ \mu\text{m K} \;/\; 9.3\ \mu\text{m} = 310\ \text{K}$$
$$T_2 = 2900\ \mu\text{m K} \;/\; 10.7\ \mu\text{m} = 270\ \text{K}$$

Now convert these temperatures into more familiar values. I gave conversion formulae during lecture on July 16:

$$T_C = T_K \ - \ 273$$
$$T_{C(1)} = T_{K(1)} \ - \ 273 = 310 \ - \ 273 = 37^\circ\ \text{C}$$
$$T_{C(2)} = T_{K(2)} \ - \ 273 = 270 \ - \ 273 = 3^\circ\ \text{C}$$

$$T_F = \frac{9}{5}\, T_K \ - \ 459.4$$
$$T_{F(1)} = \frac{9}{5}\, T_{K(1)} \ - \ 459.4 = \frac{9 \times 310\ \text{K}}{5} \ - \ 459.4 = 99^\circ\ \text{F}$$
$$T_{F(2)} = \frac{9}{5}\, T_{K(2)} \ - \ 459.4 = \frac{9 \times 270\ \text{K}}{5} \ - \ 459.4 = 27^\circ\ \text{F}$$

These numbers mean that Cat 1 has a temperature close to human body temperature, and Cat 2 is very cold, perhaps frozen. So Cat 1 may be alive, but Cat 2 is dead.