Notes on special relativity
Astronomy 250: Cosmology
Spring 1999

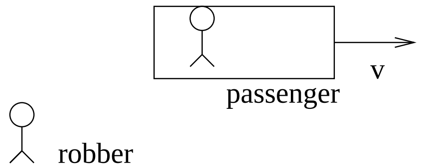
Some Notes on Special Relativity

Postulates:

1. The laws of nature are the same in all inertial frames
2. The speed of light in vacuum, c, is the same in all frames

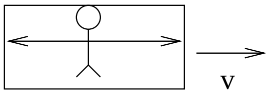
(c = 3.0 x 10^8 meters/second = 6.7 x 10^8 miles per hour)

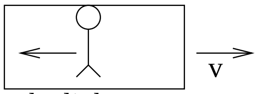
Thought experiment was passenger on train going with velocity v and robber on ground.



Simultaneity :frame dependent

The passenger can send a beam of light to hit the sides of the train at the same time



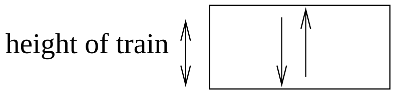


...but the robber sees the back of the train catching up with the light--the light hits the back of the train first

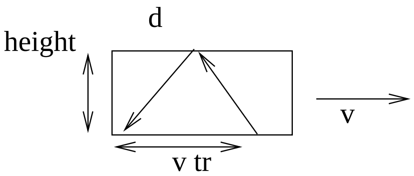
Time dilation : notion of how much time has passed is frame dependent

Passenger bounces light ray up and down.

$$\text{Velocity of light} = c = \frac{2 \text{ x height of train}}{tp}$$



Train robber sees
light travel with train.



$$\text{Velocity of light} = c = \frac{2\,d}{tr}$$

Relate d, v tr and height using pythagoras

d^2 = (v tr/2)^2 + height^2

plug in from above : d = c tr/2

height = c tp/2

(c tr/2)^2 = (v tr/2)^2 + (c tp/2)^2

or

$$tp = \frac{1}{\sqrt{1 - v^2/c^2}}\; tr$$

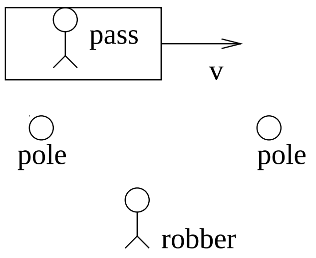
As v/c < 1, tp > tr: moving clocks run slow

You can repeat the construction with the robber shining the light up and down and the passenger measuring time elapsed and comparing.

Both think that the other one's clock is running slow and that theirs is fine-- there is no preferred frame, all inertial frames have the same laws of nature.

Length Contraction

Consider the train passing 2 telephone poles on the ground:



Passenger measures when front of train passes each pole, these are separated for passenger by time tp.

Robber measures when front of train passes each pole, and finds that this takes time tr.

The distance between the poles (length) in each frame is:

length in passenger frame = v tp

length in robber frame = v tr

So: $\frac{\text{length for passenger}}{\text{length for robber}} = \frac{tp}{tr} = \sqrt{1-v^2/c^2}$

The passenger's time is less than that for the robber, and the length the passenger sees is shorter by the same amount.

"length contraction"

The time measured between two events is the longest in the frame where the observer present at both events is at rest.

Note there is a contradiction if one tries to have length contraction in a direction not along the line of travel (eg train in tunnel--seems to crash according to one observer but not the other)

Spacetime

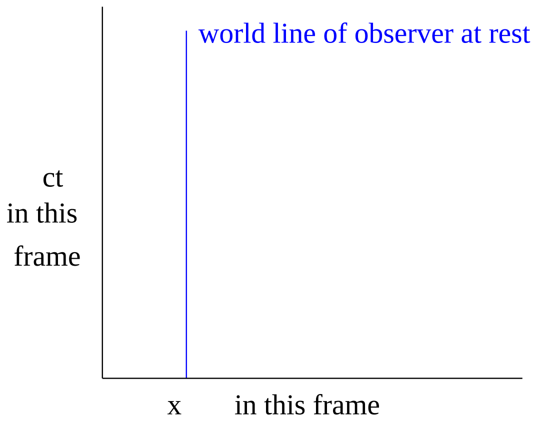
So observers cannot agree on the time passed, simultaneity or lengths.

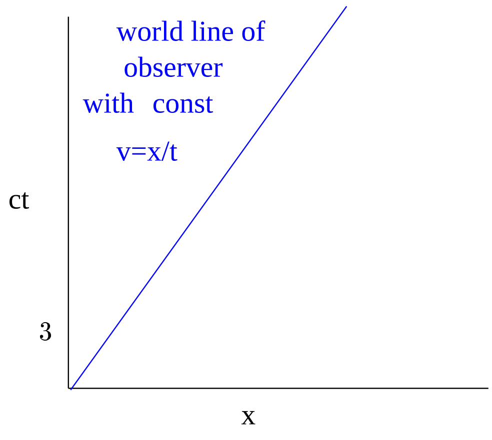
What can they agree on?

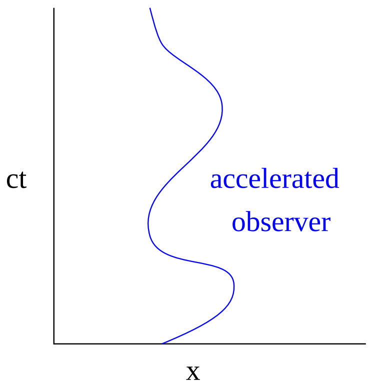
They can agree that events take place in spacetime: given by position (x,y,z) and time t.

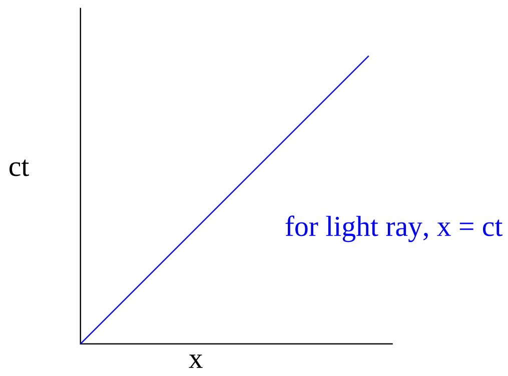
They have different coordinate systems so they measure different changes in (x,y,z) and changes in t.

Draw a path of an observer through spacetime.



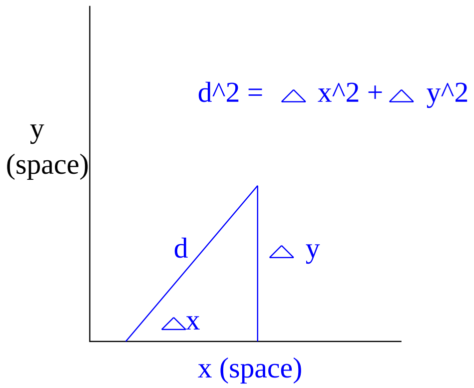




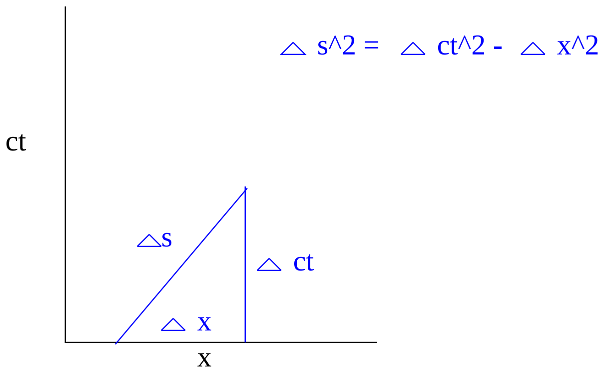


Lengths along world lines

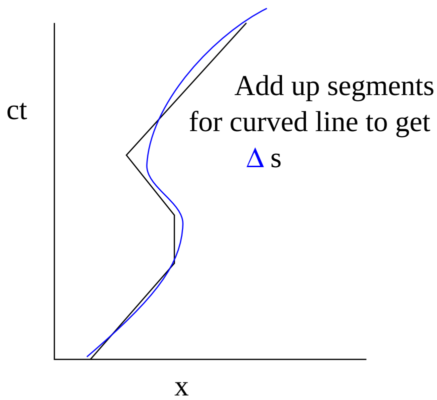
on a flat piece of paper, lengths are measured using pythagoras:



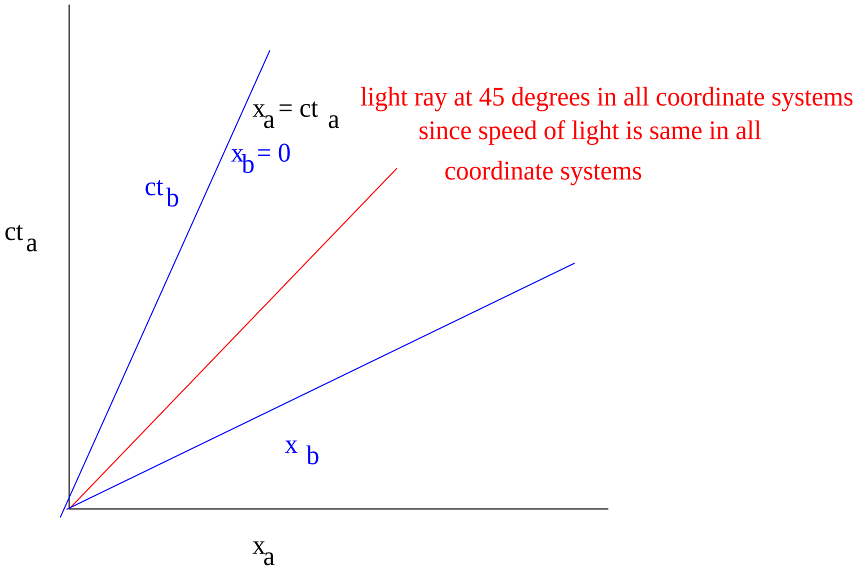
In spacetime the ’interval’ one measures is instead (Minkowski)

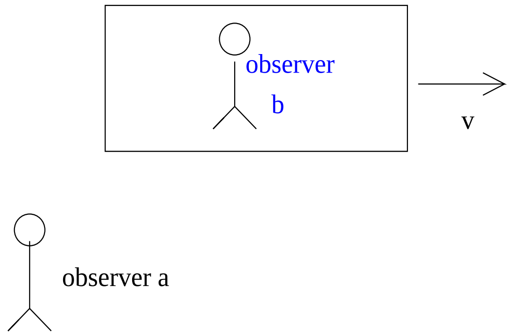


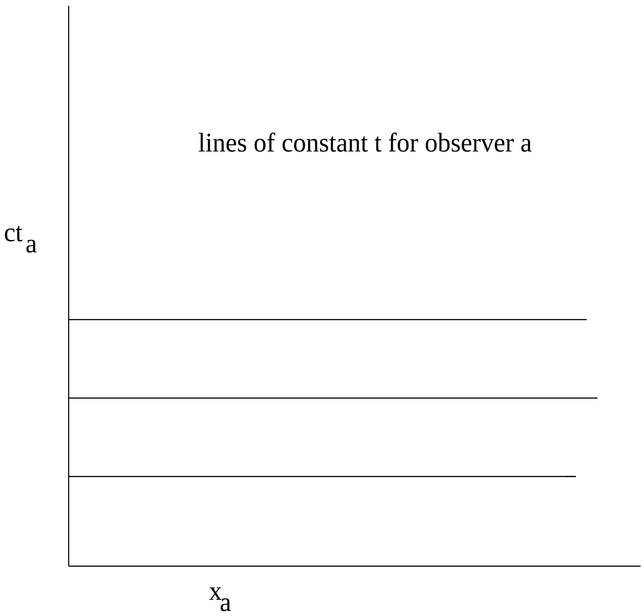
Observers in different coordinate systems agree on Δs even though they may not agree on Δx or $c\,\Delta t$.

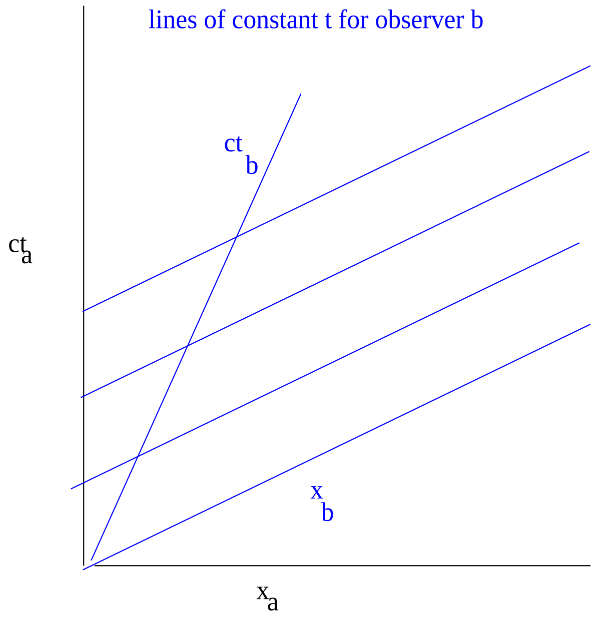


Compare coordinate systems in two time frames:









Example of how simultaneity differs in different frames:

person a on ground stands in center between two poles

and shines light outwards