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Neutrinos and Structure Formation in the Universe

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Abstract

I review the standard theory of structure formation in the Universe, starting from the assumptions that (i) the Universe is globally described by a Friedmann-Lemaître model, (ii) structure forms via gravitational instability from primordial density fluctuations, and (iii) these primordial density fluctuations were Gaussian in nature. I describe how density fluctuations grow in time, what the density perturbation power spectrum looks like, and why hot dark matter like neutrinos has a considerable impact on the process of structure formation.

Introduction

The simplest and most widely accepted theory of structure formation in the Universe starts from the following three assumptions:

1. Globally, the Universe is well described by the Friedmann-Lemaître-Robertson-Walker (FLRW) model.
2. Structure formed via gravitational instability from primordial density fluctuations.
3. These primordial density fluctuations were Gaussian in nature.

By the first assumption, the global dynamics of the Universe is described in terms of four parameters, viz. the density of ordinary and relativistic matter, the Hubble constant, and the cosmological constant. The second assumption, together with the dynamics implied by the first, specifies how structures grow in time. The third assumption asserts that the statistics of the density fluctuation field is completely specified by its mean and variance. By definition, the mean density *contrast* is zero, and the variance is specified by the *power spectrum*,

$$P(k) = \left\langle |\delta^2(\vec{k})| \right\rangle \;, \tag{1}$$

where $\vec{k}$ is the wave vector of the density perturbation. The power spectrum itself does not depend on the direction of $\vec{k}$ because of the (assumed) isotropy of the density fluctuation field.

These three basic assumptions provide the structure of this article. I will first describe the global dynamics of FLRW models, then discuss how density fluctuations grow in an expanding universe, and finally describe the gross features of the density fluctuation power spectrum, before I come to the conclusions.

Global Dynamics

The scale factor a of a FLRW universe is governed by Friedmann's equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K\,c^2}{a^2} \;, \tag{2}$$

where I have ignored the cosmological constant for simplicity. Neglecting the curvature term (as one can do at early times, see below), and approximating $\dot{a} \sim a/t$, eq. (2) implies the familiar result that the expansion time scale $t \propto \rho^{-1/2}$. The scale factor a grows monotonically with the cosmic time t and can therefore be used as a time variable. Boundary conditions for (2) are chosen such that $a(t_0) = 1$ at the present cosmic time t_0, and $t = 0$ for $a = 0$. The present value of the Hubble parameter, $H = \dot{a}/a$, is called the Hubble constant H_0. K is the curvature of spatial hypersurfaces of space-time, and ρ is the total matter density. The quantity

$$\rho_{\rm cr} \equiv \frac{3\,H_0^2}{8\pi G} \approx 2 \times 10^{-29}\,h^2\,{\rm g\,cm^{-3}} \tag{3}$$

is called the critical density, for reasons that will become clear below. The density parameter Ω_0 is the current total matter density $\rho(t_0) = \rho_0$ in units of $\rho_{\rm cr}$,

$$\Omega_0 \equiv \frac{\rho_0}{\rho_{\rm cr}}\ . \tag{4}$$

The Hubble constant is commonly written as

$$H_0 = 100\,h\,{\rm km\,s^{-1}\,Mpc^{-1}} \approx 3.2 \times 10^{-18}\,h\,{\rm s^{-1}}\ , \tag{5}$$

where $0.5 \le h \le 1$ expresses our ignorance of H_0. Note that the Hubble parameter has the dimension $({\rm time})^{-1}$, so that H^{-1} provides the natural time scale for the expansion of the Universe. Evaluating Friedmann's equation (2) at time t_0, it follows

$$K = \left(\frac{c}{H_0}\right)^2 (\Omega_0 - 1)\ . \tag{6}$$

In other words, a universe which contains the critical matter density is spatially flat.

The first law of thermodynamics, ${\rm d}U + p{\rm d}V = 0$, can be written as

$$\frac{{\rm d}(\rho c^2 a^3)}{{\rm d}t} + p\frac{{\rm d}(a^3)}{{\rm d}t} = 0\ . \tag{7}$$

For ordinary matter, $\rho c^2 \gg p \sim 0$, hence $\rho \propto a^{-3}$. For relativistic matter, $p = \rho c^2/3$, hence $\rho \propto a^{-4}$. Therefore, the matter density changes with a as

$$\rho(a) = \rho_0\,a^{-n} = \rho_{\rm cr}\,\Omega_0\,a^{-n}\ , \tag{8}$$

where $n = 3$ for ordinary matter ("dust"), and $n = 4$ for relativistic matter ("radiation").

Summarising, we can cast Friedmann's equation into the form,

$$H^2(a) = H_0^2\left[\Omega_0\,a^{-n} - (\Omega_0 - 1)\,a^{-2}\right]\ . \tag{9}$$

Since the universe expands, $a < 1$ for $t < t_0$, and so the expansion rate $H(a)$ was larger in the past. At very early times, $a \ll 1$, the first term in eq. (9) dominates because $n \ge 3$, and we can write

$$H(a) = H_0\,\Omega^{1/2}\,a^{-n/2}\ . \tag{10}$$

This is called the Einstein-de Sitter limit of Friedmann's equation.

Photon and Neutrino Backgrounds

The Universe is filled with a sea of radiation. It has the most perfect black body spectrum ever measured, with a temperature of $T_{\gamma,0} = 2.73\,\mathrm{K}$. Apart from local perturbations and kinematic effects, this *Cosmic Microwave Background* (CMB) is isotropic to about one part in 10^5. It is a relic of the photons that were produced in thermal equilibrium in the early Universe. The energy density in the CMB is

$$u_{\gamma,0} = 2\,\frac{\pi^2}{30}\,\frac{(kT_{\gamma,0})^4}{(\hbar c)^3}\;, \tag{11}$$

and the equivalent matter density is $\rho_{\gamma,0} = u_{\gamma,0}/c^2$. We have seen before that the matter density in radiation changes with the scale factor like a^{-4}, hence

$$T_\gamma(a) = a^{-1}\,T_{\gamma,0}\;. \tag{12}$$

Converting the matter density in the CMB to the density parameter as in eq. (4), we find

$$\Omega_{\gamma,0} = \frac{u_{\gamma,0}}{c^2\rho_{\mathrm{cr}}} = 2.4\times 10^{-5}\,h^{-2}\;. \tag{13}$$

The energy density in radiation today is therefore about five orders of magnitude less than that in ordinary matter.

Like photons, neutrinos were produced in thermal equilibrium in the early universe. Because of their weak interaction, they decoupled from the plasma at very early times when the temperature of the universe was $kT \sim 1\,\mathrm{MeV}$. Electrons and positrons remained in equilibrium until the temperature dropped below their rest-mass energy, $kT \sim 0.5\,\mathrm{MeV}$. The consequent annihilation of electron-positron pairs heated the photons, but not the neutrinos since they had already decoupled. Therefore, the entropy of the electron-positron pairs S_{e} was completely dumped into the entropy of the photon sea S_γ. Hence,

$$(S_{\mathrm{e}} + S_\gamma)_{\mathrm{before}} = (S_\gamma)_{\mathrm{after}}\;, \tag{14}$$

where 'before' and 'after' refer to the electron-positron annihilation time. Ignoring constant factors, the entropy per particle species is $S \propto gT^3$, where g is the statistical weight of the species. For bosons, $g = 1$, and for fermions, $g = 7/8$ per spin state. Before the annihilation, we thus have the total statistical weight $g_{\mathrm{before}} = 4\cdot 7/8 + 2 = 11/2$, while after the annihilation, $g_{\mathrm{after}} = 2$ because only photons remain. From eq. (14),

$$\left(\frac{T_{\mathrm{after}}}{T_{\mathrm{before}}}\right)^3 = \frac{g_{\mathrm{before}}}{g_{\mathrm{after}}} = \frac{11}{4}\;. \tag{15}$$

After the electron-positron annihilation, the neutrino temperature is therefore lower than the photon temperature by the factor $(4/11)^{1/3}$. In particular, the neutrino temperature today is

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0} = 1.95\,\mathrm{K}\;. \tag{16}$$

Although the neutrinos are long out of thermal equilibrium, their distribution function remained unchanged since they decoupled, except that their temperature gradually dropped.

We can therefore calculate their energy density as for the photons, and convert this to the equivalent cosmic density parameter. The result is

$$\Omega_{\nu,0} = \frac{u_{\nu,0}}{c^2 \rho_{\rm cr}} = 2.8 \times 10^{-6}\, h^{-2} \tag{17}$$

per neutrino species. Likewise, their number density is given by the Fermi distribution with temperature $T_{\nu,0}$,

$$n_{\nu,0} = \frac{3\zeta(3)}{2\pi^2} \left(\frac{kT_{\nu,0}}{\hbar c}\right)^3 \approx 113\,{\rm cm}^{-3} \tag{18}$$

per neutrino species. Suppose now there is one neutrino species which has rest mass $m_\nu > 0$, while the other species are massless. For the dark matter to be dominated by neutrinos, m_ν needs to satisfy

$$m_\nu\, n_{\nu,0} \overset{!}{=} \Omega_0\, \rho_{\rm cr} \;, \tag{19}$$

hence,

$$m_\nu\, c^2 \simeq 95\,{\rm eV}\,(\Omega_0 h^2) \;. \tag{20}$$

Adding the energy density of the two massless neutrino species to that of the photons, the total equivalent mass density in relativistic particles is

$$\Omega_{\rm R,0} = 3.0 \times 10^{-5}\, h^{-2} \;. \tag{21}$$

The Horizon

The size of causally connected regions of the Universe is called the *horizon size*. It is given by the distance a photon can travel in the time since the Big Bang. Since the appropriate time scale is provided by the inverse Hubble parameter, the horizon size is $d'_{\rm H} = c\, H(a)^{-1}$, and the *comoving* horizon size is

$$d_{\rm H}(a) = \frac{c}{a\, H(a)} = \frac{c}{H_0}\, \Omega^{-1/2}\, a^{n/2-1} \;, \tag{22}$$

where we have inserted the Einstein-de Sitter limit (10) of Friedmann's equation. $c\, H_0^{-1} = 3\, h^{-1}\,{\rm Gpc}$ is called the *Hubble radius*.

The previous calculations show that the matter density today is completely dominated by ordinary rather than relativistic matter. But since the relativistic matter density grows faster with decreasing scale factor a than the ordinary matter density, there had to be a time $a_{\rm eq} \ll 1$ before which relativistic matter dominated. The condition

$$\Omega_{\rm R,0}\, a_{\rm eq}^{-4} \overset{!}{=} \Omega_0\, a_{\rm eq}^{-3} \tag{23}$$

yields

$$a_{\rm eq} = 3.0 \times 10^{-5}\, (\Omega_0 h^2)^{-1} \;. \tag{24}$$

We shall see later that the horizon size at the time $a_{\rm eq}$ plays a very important rôle for structure formation. Under the simplifying assumption that matter dominated completely for all $a \geq a_{\rm eq}$ (i.e. ignoring the contribution from radiation to the matter density), eq. (22) yields

$$d_{\rm H}(a_{\rm eq}) = \frac{c}{H_0}\, \Omega_0^{-1/2}\, a_{\rm eq}^{1/2} \approx 13\, (\Omega_0\, h^2)^{-1}\,{\rm Mpc} \;. \tag{25}$$

The (photon) temperature at $a_{\rm eq}$ is

$$kT_\gamma(a_{\rm eq}) = a_{\rm eq}^{-1}\, kT_{\gamma,0} \simeq 8\,{\rm eV}\ . \tag{26}$$

This is much lower than the rest-mass energy of a cosmologically relevant neutrino. Hence, if there is a neutrino species with mass given by eq. (20), it becomes non-relativistic much earlier than $a_{\rm eq}$.

Growth of Density Fluctuations

a) Linear Growth

Imagine a FLRW background model into which a spherical, homogeneous region is embedded which has a slightly different density than the surroundings. Because of the spherical symmetry, this sphere will evolve like a universe of its own; in particular, it will obey Friedmann's equation. Let indices 0 and 1 denote quantities within the surrounding universe and the perturbed region, respectively. Then, Friedmann's equation reads

$$\begin{aligned} H_1^2 + \frac{Kc^2}{a_1^2} &= \frac{8\pi G}{3}\,\rho_1 \\ H_0^2 &= \frac{8\pi G}{3}\,\rho_0 \end{aligned} \tag{27}$$

for the perturbation and for the surrounding universe, respectively. We have ignored the curvature term in the second equation because the background has $K = 0$ at early times.

We now require that the perturbed region and the surrounding universe expand at the same rate, hence

$$H_0 \stackrel{!}{=} H_1\ . \tag{28}$$

Assuming that the density inside the perturbed region only weakly deviates from that of the background, we can put $a_1 \simeq a_0$, and eqs. (27) and (28) yield

$$\delta \equiv \frac{\rho_1 - \rho_0}{\rho_0} = \frac{3\,Kc^2}{8\pi G}\,\frac{1}{\rho_0 a^2} \propto (\rho_0 a^2)^{-1}\ . \tag{29}$$

We have seen before that $\rho_0 \propto a^{-n}$, with $n = 4$ before $a_{\rm eq}$ and $n = 3$ thereafter. It thus follows that

$$\delta \propto \begin{cases} a^2 & \text{before } a_{\rm eq}, \\ a & \text{after } a_{\rm eq}. \end{cases} \tag{30}$$

This heuristic result is confirmed by accurate, relativistic and non-relativistic perturbation calculations.

b) Suppression of Growth

A perturbation of wavelength λ is said to "enter the horizon" when $\lambda = d_{\rm H}$. If $\lambda < d_{\rm H}(a_{\rm eq})$, the perturbation enters the horizon while radiation still dominates. Until $a_{\rm eq}$, the expansion time scale, $t_{\rm exp}$, is determined by the radiation density $\rho_{\rm R}$, and therefore it is shorter than the collapse time scale of the dark matter, $t_{\rm DM}$:

$$t_{\rm exp} \sim (G\rho_{\rm R})^{-1/2} < (G\rho_{\rm DM})^{-1/2} \sim t_{\rm DM}\ . \tag{31}$$

In other words, the fast radiation-driven expansion prevents dark-matter density perturbations from collapsing. Light can only cross regions that are smaller than the horizon size. The suppression of growth due to radiation is therefore restricted to scales smaller than the horizon, and larger-scale perturbations remain unaffected. This suggests that the horizon size at $a_{\rm eq}$, $d_{\rm H}(a_{\rm eq})$, sets an important scale for structure growth.

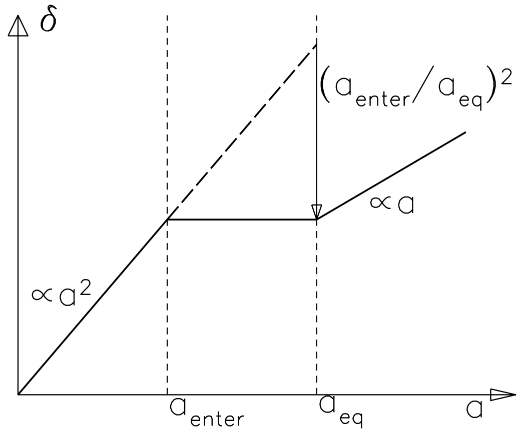

Figure 1: Sketch illustrating the suppression of structure growth during the radiation-dominated phase. The perturbation grows $\propto a^2$ before $a_{\rm eq}$, and $\propto a$ thereafter. If the perturbation is smaller than the horizon at $a_{\rm eq}$, it enters the horizon at $a_{\rm enter} < a_{\rm eq}$ while radiation is still dominating. The rapid radiation-driven expansion prevents the perturbation from growing further. Hence it stalls until $a_{\rm eq}$. By then, its amplitude is smaller by $f_{\rm sup} = (a_{\rm enter}/a_{\rm eq})^2$ than it would be without suppression.

Figure 1 illustrates the growth of a perturbation with $\lambda < d_{\rm H}(a_{\rm eq})$, that is small enough to enter the horizon at $a_{\rm enter} < a_{\rm eq}$. It can be read off from the figure that such perturbations are suppressed by the factor

$$f_{\rm sup} = \left(\frac{a_{\rm enter}}{a_{\rm eq}}\right)^2 . \tag{32}$$

According to eq. (22), the comoving horizon size scales with a as $d_{\rm H} \propto a^{n/2-1}$. When a perturbation enters the horizon, $\lambda = d_{\rm H}(a_{\rm enter})$, which yields $a_{\rm enter} \propto \lambda^{2/(n-2)} = k^{-2/(n-2)}$, or $a_{\rm enter} \propto \lambda = k^{-1}$ during the radiation-dominated epoch. Thus we obtain from (32)

$$f_{\rm sup} = \left(\frac{\lambda}{d_{\rm H}(a_{\rm eq})}\right)^2 = \left(\frac{k_0}{k}\right)^2 , \tag{33}$$

where k_0 is the wave number corresponding to $d_{\rm H}(a_{\rm eq})$.

The Perturbation Spectrum

Consider now the primordial perturbation spectrum at very early times, $P_{\rm i}(k) = |\delta_{\rm i}^2(k)|$. Since the density contrast grows as $\delta \propto a^{n-2}$, the spectrum grows as $P(k) \propto a^{2(n-2)}$. At $a_{\rm enter}$, the spectrum has therefore changed to

$$P_{\mathrm{enter}}(k) = a_{\mathrm{enter}}^{2(n-2)}\, P_{\mathrm{i}}(k) \propto k^{-4}\, P_{\mathrm{i}}(k)\ . \tag{34}$$

It is commonly assumed that the total power of the density fluctuations at a_{enter} should be scale-invariant. This implies $k^3 P_{\mathrm{enter}}(k) = \mathrm{const.}$, or $P_{\mathrm{enter}}(k) \propto k^{-3}$. Accordingly, the primordial spectrum has to scale with k as $P_{\mathrm{i}}(k) \propto k$. This *scale-invariant* spectrum is called the Harrison-Zel'dovich-Peebles spectrum. Combining that with the suppression of small-scale modes, we arrive at

$$P(k) \propto \begin{cases} k & \text{for } k < k_0, \\ k^{-3} & \text{for } k > k_0. \end{cases} \tag{35}$$

Damping by Free Streaming

If the particles of the dominant dark-matter component are fast enough ("hot"), perturbations can be washed out by free streaming. To see this, consider the so-called *free-streaming length* λ_{FS}, that is the length that particles can cover within the available time t. It is given by

$$\lambda_{\mathrm{FS}}(t) = a(t) \int_0^t \frac{v(t')}{a(t')}\, \mathrm{d}t'\ . \tag{36}$$

While particles are relativistic, $v = c$, and later $v \propto a^{-1}$ because the momentum is red-shifted. We thus introduce a new scale, the scale factor a_{nr} where the dark-matter particles become non-relativistic. It is defined by the condition $kT = a_{\mathrm{nr}}^{-1} kT_{\gamma,0} \simeq m_\nu c^2$. Inserting $T_{\gamma,0} = 2.73\,\mathrm{K}$ and the neutrino mass m_ν from eq. (20), we find

$$a_{\mathrm{nr}} = 4.7 \times 10^{-6}\, (\Omega_0 h^2)^{-1}\ . \tag{37}$$

Evaluating eq. (36), the *comoving* free-streaming length turns out to be

$$\lambda_{\mathrm{FS}} = \frac{2ct_{\mathrm{nr}}}{a_{\mathrm{nr}}} \begin{cases} \frac{a}{a_{\mathrm{nr}}} & \text{for } a < a_{\mathrm{nr}}, \\ 1 + \ln\left(\frac{a}{a_{\mathrm{nr}}}\right) & \text{for } a_{\mathrm{nr}} < a < a_{\mathrm{eq}}, \\ \frac{5}{2} + \ln\left(\frac{a_{\mathrm{eq}}}{a_{\mathrm{nr}}}\right) - \frac{3}{2}\left(\frac{a_{\mathrm{eq}}}{a}\right)^{1/2} & \text{for } a_{\mathrm{eq}} < a, \end{cases} \tag{38}$$

where λ_{FS} is plotted in Fig. 2 as a function of a. This figure shows that λ_{FS} quickly reaches an asymptotic value $\lambda_{\mathrm{FS}}^{\infty}$. For dark matter dominated by neutrinos, this asymptotic free-streaming length is

$$\lambda_{\mathrm{FS}}^{\infty} = 11.3\,\mathrm{Mpc}\, (\Omega_0 h^2)^{-1}\ . \tag{39}$$

We therefore conclude that if neutrinos were to dominate the dark matter, fluctuations with $\lambda \le \lambda_{\mathrm{FS}}^{\infty}$ would be wiped out by free streaming. Such perturbations cannot keep the fast neutrinos bound. Consequently, the power spectrum is exponentially cut off at wave numbers $k \ge k_{\mathrm{FS}} = (\lambda_{\mathrm{FS}}^{\infty})^{-1}$. Since k_{FS} is only slightly larger than the wave number where the power spectrum reaches its peak, k_0, the neutrino-dominated power spectrum is cut off right above the peak. This is illustrated in Fig. 3 which contrasts cold with hot dark matter spectra. For cold dark matter (CDM), free streaming is unimportant, so the spectrum behaves as described by eq. (35). For hot dark matter (HDM), the spectrum is cut off exponentially at large

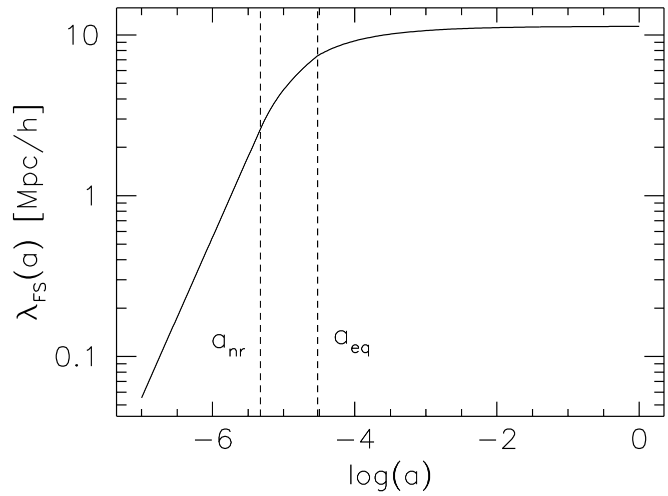

Figure 2: The comoving free-streaming length $\lambda_{\rm FS}(a)$ as a function of the scale factor a. $\lambda_{\rm FS}$ grows rapidly until $a_{\rm nr}$ and quickly approaches an asymptotic value $\lambda_{\rm FS}^{\infty}$ thereafter.

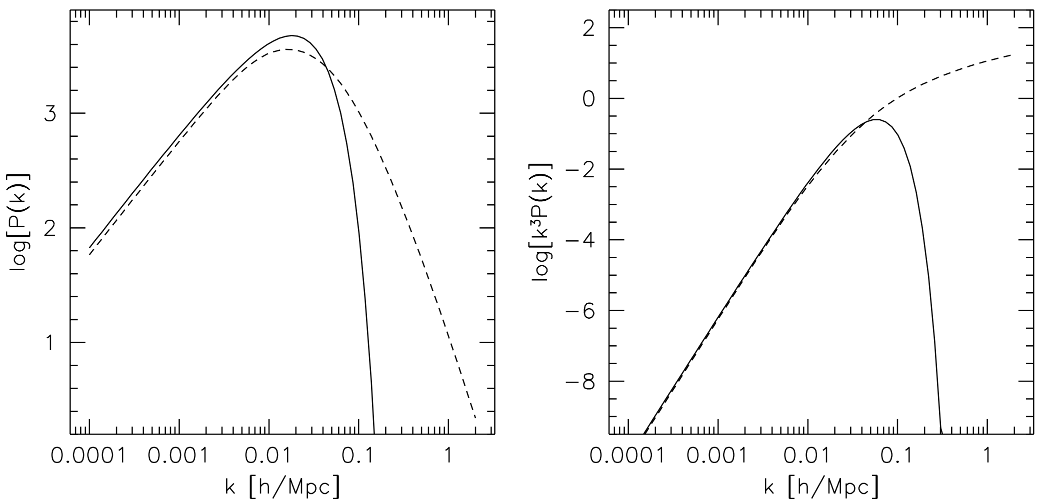

Figure 3: Left panel: Power spectrum $P(k)$ for cold and hot dark matter (dashed and solid lines, respectively). For cold dark matter, the spectrum falls off $\propto k^{-3}$ at large k, while free streaming of hot dark matter like neutrinos exponentially damps away perturbations smaller than the asymptotic free-streaming length $\lambda_{\rm FS}^{\infty}$. Right panel: Total power $k^3P(k)$ for cold and hot dark matter. While the power continues to increase monotonically for $k \to \infty$ for cold dark matter (solid curve), it reaches a peak at $k = k_0$ for hot dark matter and decreases rapidly on smaller scales.

k. Consequently, the total power of the density fluctuations, $k^3P(k)$, grows monotonically with k for CDM, while it peaks around k_0 for HDM. Structure begins to form on scales where the power is largest. For CDM, the smallest structures are formed first, and structure formation proceeds towards larger scales. For HDM, structures on scales of $\lambda \sim k_0^{-1}$ form first. They correspond to objects somewhat more massive than galaxy clusters. Smaller structures

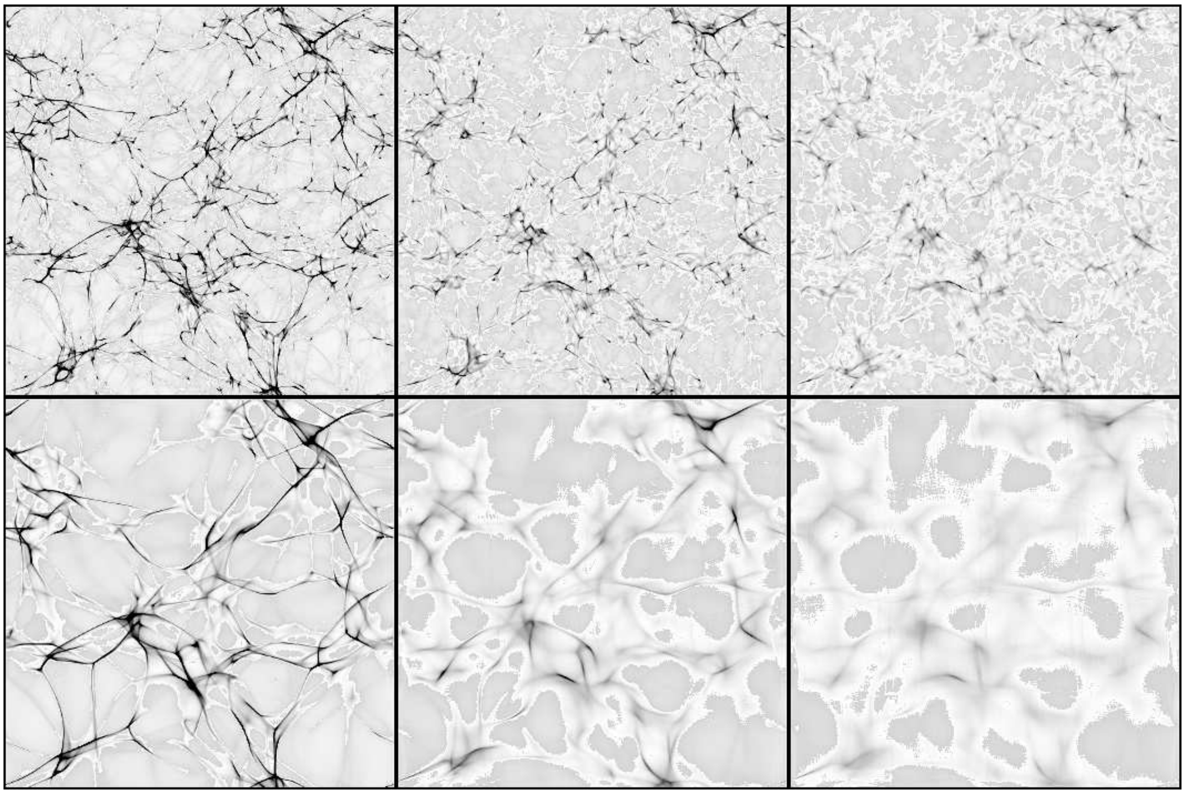

Figure 4: Growth of structure in CDM (top) and HDM (bottom) from the same initial random density perturbations; from left to right: $z = 0$, $z = 1$, and $z = 2$. High-density regions are black.

form later by fragmentation. Thus, free streaming has the important consequence that the *bottom-up* scenario of CDM is changed to the *top-down* scenario of HDM. The formation of structures smaller than galaxy clusters, e.g. galaxies, is therefore strongly delayed in HDM models. Figure 4 illustrates this. It shows two cosmological simulations starting from the same initial random density perturbations, one with CDM and one with HDM. Three epochs of the evolution at $z = 2, 1, 0$ are shown. While significant structure on small scales is already seen at $z = 2$ in CDM, structure then just begins to grow on much larger scales in HDM.

Summary

The main results of the preceding discussion can be summarised as follows:

- The matter density of the Universe today is dominated by ordinary rather than relativistic matter. Going back in time, the radiation density grows faster than the matter density. The epoch when they equal each other plays an important rôle for structure formation. It is characterised by the scale factor

$$a_{\mathrm{eq}} = 3.0 \times 10^{-5} \, (\Omega_0 h^2)^{-1} \; .$$

- Linear density perturbations grow $\propto a^2$ before a_{eq}, and $\propto a$ thereafter.

- Rapid radiation-driven expansion earlier than $a_{\rm eq}$ suppresses the growth of modes which are smaller than the size of the horizon at $a_{\rm eq}$,

$$d_{\rm H}(a_{\rm eq}) \sim 13\,(\Omega_0 h^2)^{-1}\,{\rm Mpc}\ .$$

  Density perturbations with wave number $k > d_{\rm H}^{-1}(a_{\rm eq}) = k_0$ are suppressed by the factor

$$f_{\rm sup} = \left(\frac{k_0}{k}\right)^4\ .$$

  Together with the assumption of a scale-free primordial power spectrum, this yields the cold dark matter spectrum

$$P(k) \propto \left\{ \begin{array}{ll} k & (k < k_0), \\ k^{-3} & (k > k_0). \end{array} \right.$$

- Free streaming of hot dark-matter particles like neutrinos erases structures smaller than the asymptotic, comoving free streaming length

$$\lambda_{\rm FS}^{\infty} \sim 11\,(\Omega_0 h^2)^{-1}\,{\rm Mpc}\ .$$

  The hot dark matter spectrum therefore decreases exponentially for $k > (\lambda_{\rm FS}^{\infty})^{-1}$.

- This implies that structure formation in a neutrino-dominated universe proceeds according to the top-down scenario: small structures like galaxies form later than large structures like galaxy clusters, in conflict with observational results.

Note: There is a wealth of literature on the subject briefly reviewed here. For further reading, and for references on details, I can particularly recommend the following text-books:

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