Physics 137A/2, Spring 2002

Properties of Hermite Polynomials

The Hermite polynomial $H_n(x)$ is of order n and of even parity if n is even and odd parity if n is odd. We can define $H_n(z)$ as

$$H_n(z) \equiv (-1)^n e^{z^2} \frac{d^n}{dz^n} e^{-z^2} \tag{1}$$

from which it is easy to show

$$H_n(z) = \left(2z - \frac{d}{dz}\right) H_{n-1}(z) \tag{2}$$

They can be generated by

$$e^{-\lambda^2+2\lambda z} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} H_n(z) \tag{3}$$

from which we can derive

$$H_n(z) = \left(\frac{\partial^n}{\partial \lambda^n} e^{-\lambda^2+2\lambda z} \right)\Bigg|_{\lambda=0} \tag{4}$$

By differentiating Eq.(3) with respect to z and equating powers of λ one finds

$$H_n'(z) = 2n H_{n-1}(z) \tag{5}$$

or by differentiating Eq.(3) with respect to λ and equating powers of λ we have

$$H_n(z) = 2z H_{n-1}(z) - 2(n-1) H_{n-2}(z) \tag{6}$$

Finally we may differentiate Eq.(5) and use Eq.(2) to show

$$\left(\frac{d^2}{dz^2} - 2z \frac{d}{dz} + 2n \right) H_n(z) = 0 \tag{7}$$

The first few Hermite polynomials are

$$H_0(z) = 1 \tag{8}$$
$$H_1(z) = 2z \tag{9}$$
$$H_2(z) = 4z^2 - 2 \tag{10}$$
$$H_3(z) = 8z^3 - 12z \tag{11}$$
$$H_4(z) = 16z^4 - 48z^2 + 12 \tag{12}$$