REDSHIFT SPACE DISTORTIONS : KAISER FORMULA AND BEYOND.

LSS MINI COURSE

- INTRODUCTION
- KAISER FORMULA (SMALL ANGLE LINEAR REGIME)
- EXTENSION TO LARGE ANGLES
- NON LINEAR REGIME
- CONNECTING LINEAR AND NON LINEAR REGIMES
- OBSERVATIONS AND COSMOLOGICAL PARAMETERS
- CONNECTION WITH THE HALO MODEL

INTRODUCTION

— REDSHIFT MEASUREMENT KEY TO LARGE SCALE STRUCTURE SURVEYS

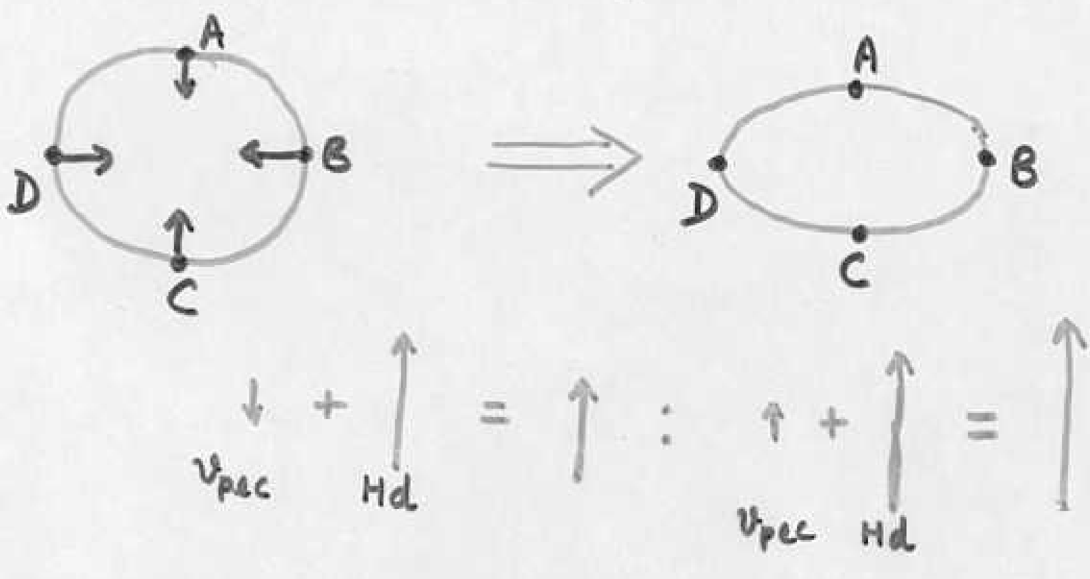
— MEASURED REDSHIFT HAS TWO COMPONENTS

- — FROM THE HUBBLE EXPANSION
- — PECULIAR MOTION OF GALAXIES

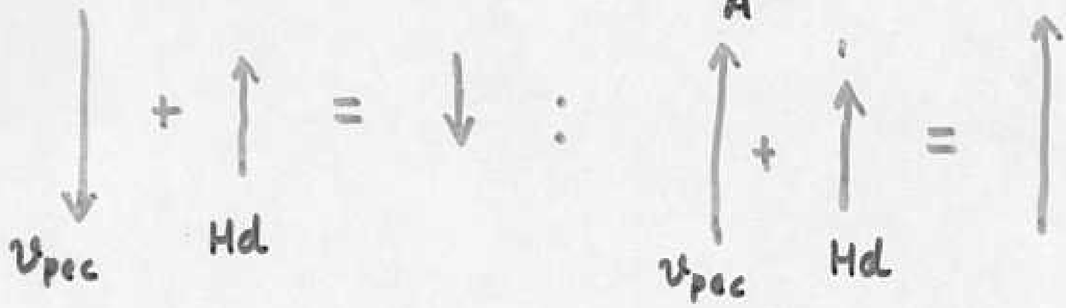
$$v_{\text{MEASURED}} = \underbrace{Hd}_{\substack{\text{REAL SPACE}\\ \text{EXPANSION}}} + \underbrace{v_{pec}}_{\substack{\text{INFO. ABOUT G-POTENTIAL}\\ \text{WELLS}}}$$

— TWO EFFECTS

- — LINEAR REGIME $v_{pec} < Hd$



— NON LINEAR REGIME



KAISER FORMULA

— LINE OF SIGHT COMPONENT OF THE PECULIAR VELOCITY, $\vec{u}(\vec{r})$

$$v = \vec{u}(\vec{r}) \cdot \hat{r}$$

— POSITION IN REDSHIFT SPACE,

$$s = r + v$$

$$\frac{ds}{dr} = 1 + \frac{\partial v}{\partial r}$$

— NUMBER OF GALAXIES IS CONSERVED,

$$n_s(s)\, d^3s = n_r(r)\, d^3r$$

$$\Rightarrow \bar{n}_s\left(1+\delta^s(s)\right) s^2 ds = \bar{n}_r(r)\left(1+\delta(r)\right) r^2 dr$$

$$\Rightarrow \left[1+\delta^s(s)\right] = \frac{\bar{n}_r(r)}{\bar{n}_s(s)} \frac{1}{\left[1+\frac{v}{r}\right]^2} \frac{1}{\left[1+\frac{\partial v}{\partial r}\right]} \left[1+\delta(r)\right]$$

— $\bar{n}_r(r) = \bar{n}_s(s)$ AND TO LINEAR ORDER,

$$\left[1+\delta^s(s)\right] \simeq \left[1+\delta(r)\right]\left(1 - \frac{\partial v}{\partial r} - \frac{2v}{r} \cdots\right)$$

— PLANE PARALLEL APPROXIMATION

$$\frac{\partial v}{\partial r} \gg \frac{2v}{r}$$

$$\Rightarrow \delta^s(s) \simeq \delta(r) - \frac{\partial v}{\partial r} \qquad \text{— } (1)$$

ALSO IN LINEAR THEORY,

$$\vec{\nabla}\cdot\vec{u} + \frac{d\delta}{dt} = 0$$

$$\Rightarrow \quad i\vec{k}\cdot\vec{u} + \delta \frac{d\ln\delta}{d\ln a}\frac{d\ln a}{dt} = 0$$

$$\Rightarrow \quad \vec{u} = \frac{i\vec{k}}{k^2}\,\delta\,\frac{d\ln\delta}{d\ln a}\cdot\frac{d\ln a}{dt}$$

DEFINE $\mu = \hat{r}\cdot\hat{k}$

$$\Rightarrow \quad v = \hat{r}\cdot\vec{u} = \frac{i\mu}{k}\,\delta\,\underbrace{\frac{d\ln\delta}{d\ln a}}_{\substack{\text{GROWTH}\\ \text{FACTOR}\\ f(\Omega)\simeq\Omega^{0.6}}}\cdot\underbrace{\frac{d\ln a}{dt}}_{H=1}$$

$$\Rightarrow \quad v = -\frac{\mu}{ki}\,\delta f(\Omega)$$

$$\Rightarrow \quad \frac{\partial v}{\partial r} = -\mu^2\delta f(\Omega) \quad - \text{②}$$

USE ① & ②

$$\Rightarrow \quad \delta^s(s) = \delta_r(r)\left[1+\mu^2 f(\Omega)\right]$$

— PUT IN THE BIAS ASSUMING IT TO BE SCALE AND REDSHIFT INVARIANT

$$P_s(k) = \left[1 + \mu^2 \beta\right]^2 P(k)$$

— EXPAND IN LEGENDRE POLYNOMIALS

$$P_s(k) = \left[\left(1 + \tfrac{2}{3}\beta + \tfrac{1}{5}\beta^2\right) P_0(\mu) + \left(\tfrac{4}{3}\beta + \tfrac{4}{7}\beta^2\right) P_2(\mu) + \tfrac{8}{35}\beta^2 P_4(\mu)\right] P(k)$$

— KAISER FORMULA TURNS OUT TO BE GOOD ON VERY LARGE SCALE (~ FEW 100 Mpc) SO NEED EXTENSIONS TO WORK WITH LSS SURVEY DATA SETS.

EXTENSION TO LARGE ANGLES

(HEAVENS & TAYLOR '95 ; SZALAY, MATSUBARA & LANDY '98)

- SMALL ANGLE APPROACH OF KAISER WORKS ONLY AT SMALL WAVELENGTHS WHERE STRUCTURES ARE NONLINEAR
- NOW WE CAN USE ALL GALAXIES IN THE SURVEY
- AT LARGE ANGLES WE ARE IN THE LINEAR REGIME
- OUTLINE OF THE METHOD
  - EXPAND REDSHIFT SPACE DENSITY FIELD IN SPHERICAL HARMONICS

$$\tilde{\rho}_{\ell mn}(s) = C_{\ell n} \int \rho(s) \underbrace{j_\ell(k_{\ell n} s)}_{\text{SPHERICAL BESSEL FNC'S}} \underbrace{Y^*_{\ell m}(\theta, \phi)}_{\text{SPHERICAL HARMONICS}} d^3 s$$

  - NUMBER CONSERVATION OF GALAXIES

$$\Rightarrow \rho(s)\, d^3 s = \rho(r)\, d^3 r$$

$$\Rightarrow \tilde{\rho}_{\ell mn} = C_{\ell n} \int \rho(r)\, j_\ell(k_{\ell n} s)\, Y^*_{\ell m}(\theta, \phi)\, d^3 r \quad -(1)$$

$$j_{\ell n}(k_{\ell n} s) = j_\ell(k_{\ell n}(r+v))$$
$$\simeq j_\ell(k_{\ell n} r) + v(r) \frac{d}{dr} j_\ell(k_{\ell n} r).$$

TO LINEAR ORDER

ALSO IN LINEAR THEORY,
$$\vec{v} = -\nabla\psi$$
$$\vec{\nabla}.\vec{v} = -\delta f(\Omega)$$
$$\Rightarrow \nabla^2\psi = \delta f(\Omega)$$
$$v(r) = \hat{r}.\vec{v}$$
$$= f(\Omega) \sum_{\ell m n} \frac{c_{\ell n}\, \delta_{\ell m n}\, j'(k_{\ell n} r)\, Y_{\ell m}(\theta, \phi)}{k_{\ell n}} \quad - (2)$$

PLUG (2) IN (1)

NON LINEAR REGIME
(PEEBLES '80)

- EXCESS PROBABILITY OF FINDING TWO GALAXIES AT s AND s' IN REDSHIFT SPACE

$$dP = \bar{n}^2 d^3s\, d^3s' \left[1 + \xi\underbrace{(r_\perp, r_\parallel)}_{}\right] \quad -① $$

ANISOTROPIC IN REDSHIFT SPACE

- MEAN VALUE OF THE RELATIVE PECULIAR VELOCI

$$\vec{v} = -g(r)\,\overleftrightarrow{r}\ ;\ g(r) \to 1 \text{ ON SMALL SCALES},\quad \to 0 \text{ ON LARGE SCALES}$$

- DISTRIBUTION OF VELOCITIES AROUND MEAN (ISOTROPIC)

$$dP_1 = F(v)\, dv$$

- IF y IS THE TRUE SEPARATION ALONG LINE OF SIGHT

$$r_\parallel = y\left[1 - g(r)\right] + v_\parallel$$

- CONJECTURE

$$dP = \bar{n}^2 d^3s\, d^3s' \left[1 + \xi(r)\right] F(v)\, \delta^D\left[r_\parallel - y - g(r)y - v_\parallel\right] dv_\parallel \quad -②$$

- EQUATE R.H.S. OF ① & ② AND PERFORM $dv_\parallel$ & dy INTEGRALS

$$\xi(r_\perp, r_\parallel) = \int_{-\infty}^{\infty} dy\, \xi(r)\, F_v\left[r_\parallel - y + g(r)y\right]$$

— ASSUME SOME FORM FOR $F(v)$ TO OBTAIN MAP

— FOR A GAUSSIAN DISTRIBUTION

$$\delta_s(k) = \delta(k)\, e^{-(k\sigma\mu)^2/2}$$

— EXPONENTIAL AND LORENTZIAN WORK WELL TOO

CONNECTING LINEAR & NONLINEAR REGIMES

- NONLINEAR FORMULATION IGNORES δv COUPLING
- ALSO ASSUMES THAT THE VELOCITY DISPERSION IS INDEPENDENT OF POSITION
- THE REGIME BETWEEN THE TWO EXTREMES IS MOST INTERESTING OBSERVATIONALLY.
- DEFINE.

$$\vec{\eta} = \begin{pmatrix} \delta(\vec{x}) \\ \delta(\vec{x}') \\ v(\vec{x}) \\ v(\vec{x}') \end{pmatrix}$$

$$\left[1+\xi(r_\perp, r_\parallel)\right] = \int d^4\vec{\eta}\, dy\, (1+\delta)(1+\delta')\, F_\eta(\vec{\eta})\, \delta^D(r_\parallel - y - v' + v)$$

- CAN DERIVE KAISER'S FORMULA FROM HERE BUT NONLINEARITY PREVENTS EXACT SOLUTION. FUTURE ......

OBSERVATIONS AND COSMOLOGICAL PARAMETERS

— USEFUL FOR EXTRACTING REAL SPACE MAPS OF GALAXIES AND COMPARISON WITH THEORIES.

— MEASURING $\beta \equiv \frac{\Omega^{0.6}}{b}$

— RATIO OF REDSHIFT SPACE TO REAL SPACE ANGLE AVERAGED POWER SPECTRA

$$\frac{P_s(k)}{P(k)} = 1 + 2\beta + \frac{1}{5}\beta^2$$

— RATIO OF QUADRAPOLE TO MONOPOLE HARMONICS OF POWER SPECTRUM

$$P_s(k) = \left[\left(1+\frac{2}{3}\beta+\frac{1}{5}\beta^2\right)P_0(\mu) + \left(\frac{4}{3}\beta+\frac{4}{7}\beta^2\right)P_2(\mu) + \frac{8}{35}\beta^2 P_4(\mu)\right]P(k)$$

$$P_0^S = \left(1+\frac{2}{3}\beta+\frac{1}{5}\beta^2\right)P(k)$$

$$P_2^S = \left(\frac{4}{3}\beta+\frac{4}{7}\beta^2\right)P(k)$$

$$P_4^S = \frac{8}{35}\beta^2 P(k)$$

$$\frac{P_2^S(k)}{P_0^S(k)} = \frac{\frac{4}{3}\beta+\frac{4}{7}\beta^2}{1+\frac{2}{3}\beta+\frac{1}{5}\beta^2}$$

- ASSUMES BIAS IS SCALE & REDSHIFT INDEPENDEN

- SOME RESULTS

  - 2dF $\beta = 0.43 \pm 0.07$
  - IRAS $\beta = 0.69^{+0.21}_{-0.19}$ (IR)
  - STROMLO-APM $\beta = 0.30^{+0.17}_{-0.15}$ (OPTICAL)

- CAN LEARN ABOUT GALAXY FORMATION

CONNECTION WITH THE HALO MODEL

- LARGE SCALE POWER DOMINATED BY INTERACTION OF HALOS — $P_{LIN}(k)$
- SMALL SCALE POWER DOMINATED BY INTERHALO CORRELATIONS — $P_{NL}(k)$
- IN REDSHIFT SPACE

$$P_s(k) = P_{LIN}(k)(1+f(\Omega)\mu)^2 + P_{NL}(k)\, e^{-(k\sigma\mu)^2/2}$$

- POWER AMPLIFIED ON LARGE SCALES DUE TO GRAVITATIONAL INFALL AND REDUCED ON SMALL SCALES DUE TO VIRIAL MOTIONS

REFERENCES

- REVIEW ON LINEAR REDSHIFT SPACE DISTORTION
  (HAMILTON, astro-ph/9708102)
- KAISER FORMULA
  (KAISER, MNRAS 227, 1)
- EXTENSION TO LARGE ANGLES
  (HEAVENS & TAYLOR, MNRAS 275, 483 ;
  SZALAY, MATSUBARA & LANDY, ApJL 498, L1)
- NONLINEAR REGIME
  (PEEBLES : LARGE SCALE STRUCTURE OF THE UNIVERSE
  SEC. 76)
- CONNECTING LINEAR AND NONLINEAR REGIMES
  (FISHER, ApJ 448, 494)
- OBSERVATIONS
  (HAMILTON, astro-ph/9708102 ; PEACOCK et al. /0103143)
- CONNECTION WITH THE HALO MODEL
  (WHITE, MNRAS 321, 1)