CARMA SUMMER SCHOOL - July 2007

Introduction to Millimeter Wavelength Interferometry

Melvyn Wright

Radio Astronomy Laboratory, University of California, Berkeley, CA, 94720

1. OUTLINE

- Day 1: Basic introduction to everything to get you going.
- Day 2-4:
    - Radio antennas, arrays, aperture synthesis.
    - Receivers and correlators.
    - Observing techniques, scheduling.
    - Calibration, data inspection, imaging.
- Day 5: Data analysis, future developments, VLBI.

2. DAY 1: INTRODUCTION

2.1. Sources of Radio Emission

- Radio astronomy is an observational science. We make images of the radio intensity

  $$I(\mathbf{s}, \nu, polarization, time)$$

  Must design instruments to measure I with sufficient resolution in $\mathbf{s}, \nu, polarization, time$, and sensitivity to measure the source characteristics.

- What we actually measure is

  $$I' = I * B + Noise$$

  where B is the instrumental response, and $Noise$ is additive noise from the radio receivers and the atmosphere.

- Data reduction is the process of obtaining I from our measurements, I'

  Must calibrate, and deconvolve the instrumental response to otain I from I'.

- Data Analysis is interpreting what I means for astronomy.

- We need to know something about the sources of radio emission in order to design our telescopes and observations.

- Some of the 1st observations in a new waveband are usually surveys to find the distribution and nature of the sources.

  Later observations study the details of individual sources, or classes of sources.

- Source characteristics, instrumentation, and observing techniques together define a matched filter to possible observations with millimeter wavelength interferometers.

  Source selection: astronomy, frequency, size, brightness sensitivity

2.2. Millimeter wavelength Radio Sources

- Astronomy from Comets to Cosmology
  - Emission mechanisms: thermal and non-thermal
- Thermal emission is in quasi-equilibrium with the physical temperature.
  - Black body - planets, asteroids, quiet sun
  - Dust - grey body: dust emisivity at millimeter wavelengths
  - Molecular lines: rotational-vibrational transitions
    - Star formation regions, molecular clouds, stellar envelopes, YSO, evolved stars
    - Galactic structure: CO and isotopes, CS, HCN, HCO+....
    - Spiral and Dwarf galaxies: stucture, gas content, rotation curves
- Non-Thermal emission is not in equilibrium with the physical temperature.
  - Relativistic electrons & magnetic fields, synchrotron radiation.
    - SN remnants
    - Radio galaxies: hot spots
    - Active galaxies: nuclei
    - Quasars, blazars, and seyfert galaxies.
    - Active sun: Solar flares.
    - Masers:

2.3. Atmospheric windows

- Atmospheric windows at optical, IR, and radio frequencies

  Earthbound astronomical observations are possible through atmospheric windows at optical, IR, and radio frequencies. (FIGURE 1)

  optical: $0.5 - 0.8\,\mu$m.

  IR: $1 - 1000\,\mu$m.

  Radio: $350\,\mu$m -60 m.

  λ mm: 0.3 mm -1 cm. (30 to 1000 GHz)

- λ mm astronomy atmospheric windows are defined by O_2 and H_2O lines into a number of bands bounded by tropospheric absorption at high freq and ionospheric absorption at low freq. (FIGURE 2).

- Away from the strong absorption lines, the atmospheric absorption is strongly dependent on atmospheric water vapor which has a scale height of about 2 km.

  Observations at higher, drier sites have less absorption.

- CARMA site characteristics. (FIGURE 3).

  CARMA altitude is 2196 m. Typical "good" conditions $\sim$ 4 mm precipitable water vapor.

  Best winter weather. $\sim$ 2 mm precipitable water vapor.

  c/f Mauna Kea at 4 km altitude often has less than 1 mm precipitable water vapor.

- CARMA observing bands.

  CARMA currently has receivers for 2 bands.

  A 3 mm band, which can be tuned from $\sim$ 75 – 115 GHz

  A 1 mm band, which can be tuned from $\sim$ 210 – 270 GHz

  In the 1 mm band the effects of atmospheric absorption are more serious and the advantages of a high, dry site are more obvious.

3. Mon 10 - 11 am: APERTURE SYNTHESIS BASICS

3.1. Intensity Units.

- Intensity units: $I(\mathbf{s}, \nu, polarization, time)$ Watts m^{-2}str^{-1}Hz^{-1}

  1 Jansky = 10^{-26} Watts m^{-2}str^{-1}Hz^{-1}

- Brightness Temperature

  for black body radiation

$$I = \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu/kT} - 1)}$$

T is the **brightness temperature** of an equivalent black body radiator.

$$h/k = 4.8\ [GHz/100]\ K$$

For $h\nu/kT << 1$, $I = 2kT/\lambda^2\ [1 - h\nu/2kT + ...]$

- Rayleigh Jeans brightness temperature, $I = 2kT_b/\lambda^2$
- Flux density

$$S = \int\ I\ \delta\Omega$$

integrated over an astronomical source.

3.2. Radio antennas: collecting area and resolution.

- **collecting area** $\sim D^2$

  We must build antennas to collect radio photons from astronomical sources.

  Radio astronomy: antennas are not used to transmit radio waves - as in radar astronomy - but it is often useful to think of antennas as transmitters.

  Amount of power we collect depends on the intensity, or brightness of radiation.

- Antenna has an effective collecting area $A(\mathbf{s})$

  For an elemental flat collector, the effective collecting area = $\delta A \times cos(\theta)$

- Power we collect, $P = \int\ I(s)\ A(s)\ \delta\Omega\ \delta\nu\ \delta A$ watts.

- Aperture efficiency

  $effective\ collecting\ area = aperture\ efficiency \times collecting\ area$

  aperture illumination, the weighted sum of electic fields across the aperture

  feed legs, subreflector blockage,

- Surface accuracy

  E- fields are summed in phase, so aperture efficiency also depends on surface roughness.

  Ruze losses reduce effective collecting area by a factor $\exp[(-(\frac{4\pi\sigma}{\lambda})^2]$

  Surface accuracy should be better than about $\lambda/16$ for reasonable aperture efficiency.

- **Resolution** $\sim \lambda/D$

- Voltage pattern

  antenna forms a weighted vector average of the E-field across the aperture $V(\mathbf{s}) = \int\ W(\mathbf{r})\ E(\mathbf{r})\ \exp(\frac{2\pi i \mathbf{r}.\mathbf{s}}{\lambda})\ \delta A$

- Forward Gain of antenna add up the E-field across the aperture in phase in direction s_0.

  $\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow$ in direction s_0.

- In other directions a phase gradient across the aperture reduces the vector sum of the E-field across the aperture.

  It is this phase gradient which gives an antenna resolution in direction.

  For a uniformly weighted rectangular aperture diameter D,

$$V(\theta) = \sin(\pi\ D\ \theta/\lambda)/(\pi\ D\ \theta/\lambda)$$

  For a uniformly weighted circular aperture diameter D,

$$V(\theta) = J_1(\pi\ D\ \theta/\lambda)/(\pi\ D\ \theta/\lambda)$$

3.3. Antenna Holography, Primary beam patterns

- The E-field distribution across the aperure is the Fourier transform of the Voltage pattern
$$W(\mathbf{r})\ E(\mathbf{r}) = \int V(\mathbf{s})\ \exp(-2\pi\ i\ \mathbf{r}.\mathbf{s}/\lambda)\ \delta\mathbf{s}$$

- We can measure the amplitude and phase of the voltage pattern by scanning one antenna across a strong radio source, or a radio transmitter, while using another antenna as a phase reference.

  The measured aperture distribution gives the illumination - the weighting of the E-field - across the aperture. The phase gives the surface error.

  Can clearly see the shadow of the feed legs and subreflector (FIGURE).

  Phase gradient across the aperture is due to pointing error.

  After removing the phase gradient and a quadratic term due to focus error, we measure a surface RMS $\sim$ 30 microns. These are very good antennas at millimeter wavelengths.

- Antenna power pattern
$$P(\mathbf{s}) = V(\mathbf{s}) \times V^{\star}(\mathbf{s})$$

  Primary beamwidth, FWHM $\sim 1.2\ \lambda/D$

  6m antenna primary beam FWHM (arcmin): 2.40 at 80 GHz; 0.83 at 230 GHz.

  10m antenna primary beam FWHM (arcmin): 1.34 at 80 GHz; 0.47 at 230 GHz.

3.4. Aperture Arrays

- **Antenna size, Resolution and Confusion.**
- An antenna provides collecting area $\eta\ D^2$, and resolution FWHM $\sim 1.2\ \lambda/D$.
- Large antennas with enough sensitivity to detect more than one source within the beam are **confusion limited**.

  often have enough collecting area, but we need more resolution

  can separate the functions of resolution and collecting area by building several smaller antennas.

  Need to add up the E-field across this distributed aperture - preserving the relative phase of the wavefront across the array of antennas.

  This is quite difficult. Not only do the electronics at each antenna need to preserve phase, but also atmospheric phase shifts distort the wavefront and we must compensate for these effects.

  The problem is rather like adaptive optics. We must keep the path lengths within $\sim\ \lambda/16$ to make an accurate telescope.
- **Surface accuracy** (FIGURE)
- **cost of large antennas** $\sim D^2\ \lambda^{-0.7}$ (FIGURE)

3.5. Need for high resolution

- Early radio astronomy discovered radiation from the Galaxy, with unresolved maxima.

- Peaks were named Cas A, Cygnus A, Sag A, Taurus A, Virgo A etc. Resolution ~ 7 degrees.

- Small optical telescope resolution, $\lambda/D \sim 5\ 10^{-5}/10\,\mathrm{cm} \sim 1$ arcsec.

- Arecibo, $\lambda/D \sim 6cm/300m \sim 36$ arcsec.

- Bonn, $\lambda/D \sim 1cm/100m \sim 20$ arcsec.

- IRAM 30m, $\lambda/D \sim 3mm/30m \sim 20$ arcsec.

- Small scale structure in radio sources

  hot spots in radio galaxies (0.1 - 1 arcsec)

  filaments in crab nebula (1 arcsec)

  IR sources, OH and masers in star formation regions (1 - 10 arcsec)

  molecular cloud structure (0.1 arcsec - several degrees)

  spiral arm structure in galaxies (1 arcsec - a few degrees)

  quasars components with $\lambda/D \sim 3\,\mathrm{mm}/3000\,\mathrm{km} \sim 0.2$ milliarcsec.

- Conclude that we need to observe structures over a wide range of angular scales.

  single antennas can image structures larger than $\sim 20''$ at 3mm.

  need effective antenna diameters ~ 1 kilometer to image arcsec structures.

- Techniques for high resolution observations

  lunar occultations

  interplanetary scintillations

  interferometers

3.6. Aperture Synthesis

- Consider aperture as set of sub-apertures each with its own E-field and phase.

$$\text{Voltage response} = \Sigma\ E_i\ cos(\omega\ t + \phi_i)$$

$$\text{Power} = (\Sigma\ E_i\ cos(\omega\ t + \phi_i))^2 = \Sigma\ E_i^2 + \Sigma\ E_i\ E_j\ cos(\phi_i - \phi_j)$$

  1st term is sum of total powers from all sub-apertures

  2nd term is cross products of all antenna pairs

  cross correlation and the relative phase contains information about wavefront across the aperture.

- If the source structure does not change, we can sample different pieces of the aperture at different times.

  must preserve the relative phase across the whole aperture to synthesize a large telescope.

- skeleton arrays, T-arrays contains all the relative spacings of square apperture.

  T-array has a different shaped beam because of different weights for each cross product, $E_i\ E_j$

3.7. Aperture synthesis imaging.

- response of a 2-element interferometer to point source.

- Earth rotation aperture synthesis

- coordinate systems: (u,v,w), u,v tracks for different arrays.

  East-west baseline

$$u = b/\lambda\ cos\ h; v = b/\lambda\ sin\ h\ cos\ \delta$$

- Mapping extended sources.

3.8. Properties of Synthesized Beam

1. Resolution $\sim \lambda/2b_{\max}$
   Note factor of 2 since $V(-u) = V^{\star}(u)$
2. Aliasing of structures $> \lambda/b_{inc}$
3. Shortest spacing problem
   Source "resolved out" if size $> \lambda/2b_{\min}$
4. c/f Field of view of antennas $\sim \lambda/D$ so shortest spacing is D, else collision.
   So, we can image/must select sources with structures between $\lambda/2D$ and $\lambda/2b_{\max}$.

	$\mathrm{d_{max}}$	$\mathrm{d_{inc}}$	something	yield
c-array	240 ns	20 ns	8"	100"
b-array	1000 ns	80 ns	2"	25"
a-array	4000 ns	320 ns	0.5"	6"

3.9. Interferometer Observations

$$V(t) = I(s)A(s - s') \exp \frac{2\pi i}{\lambda}\mathbf{b}.\mathbf{s}\,\mathrm{ds}$$

s' is pointing center
s_0 is phase tracking center
If σ is small, neglect curvature, else 3D transform.

$$V = \exp \frac{2\pi i}{\lambda}\mathbf{b}.\mathbf{s}(t) \int I(\sigma)A(s - s') \exp \frac{2\pi i}{\lambda}\mathbf{b}.\sigma \mathrm{d}\sigma$$

Instrumental terms Source structure

$a = (x, y, z)$; $b = (u, v, w)$

$$V(u, v) = \int I(x, y)A(x - x', y - y') \exp \frac{2\pi i}{\lambda}(ux + vy)\,\mathrm{d}x\mathrm{d}y$$

This is a chromatic instrument, ie need small range of λ or else you get bandwidth smearing. Use bandwidth synthesis (mfs) to obtain more Fourier components.

We only have discrete samples of V, so define a weighting function W.

$$I'(x, y) = \int W(u, v)V(u, v) \exp \frac{2\pi i}{\lambda}(ux + vy)\,\mathrm{d}x\mathrm{d}y$$

$\omega \neq 0$ where V is sampled; $\omega = 0$ where V is not sampled.

$$I'(x,y) = B(x,y) \star [I(x,y)A(x,y)]$$

where

$$B(x,y) = \int W(u,v) \exp \frac{-2\pi i}{\lambda}(ux + vy)\, \mathrm{d}u\mathrm{d}v$$

is the synthesized beam, ie

$$V(u,v) = (1,0),$$

a point source of unit amplitude. So the synthesized beam is a point source response.